Online Optimization of Power Networks

Changhong Zhao
Lingwen Gan
Steven Low
EE, CMS, Caltech

Enrique Mallada
ME, Johns Hopkins

January 2016

Key message

Large network of DERs

- Real-time optimization at scale
- Computational challenge: power flow solution

Online optimization (feedback control)

- Network solves hard problem in real time for free
- Exploit it for our optimization/control
- Naturally adapts to evolving network conditions

Examples

- Slow timescale: OPF
- Fast timescale: frequency control

Outline

Optimal power flow

- DistFlow model and ACOPF
- Online algorithm
- Analysis and simulations

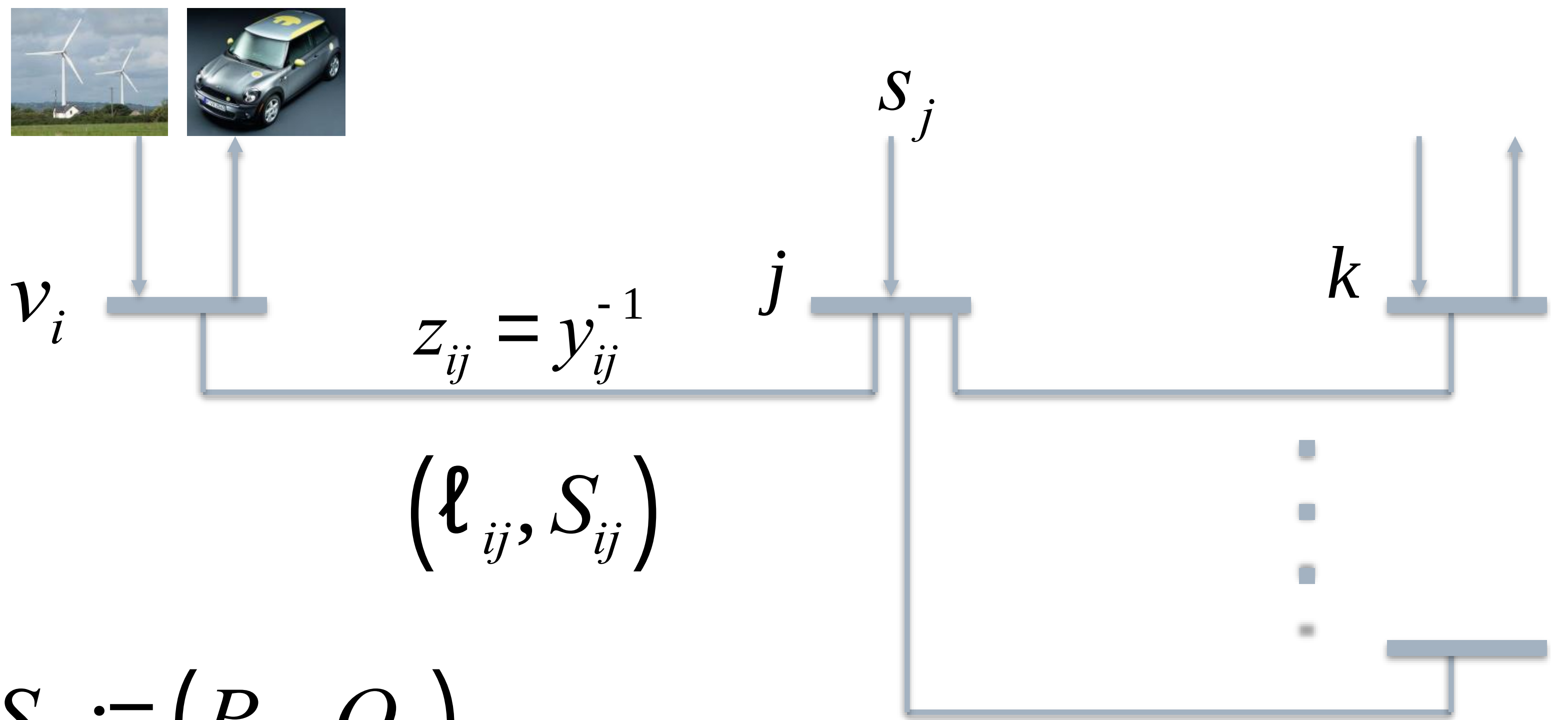
Gan & L, JSAC 2016

Load-side frequency control

- Dynamic model & design approach
- Distributed online algorithm
- Analysis and simulations
- Details

Zhao, Topcu, Li, L, TAC 2014
Mallada, Zhao, L, Allerton 2014
Zhao et al: CDC 2014, CISS 2015, PSCC 2016

Branch flow model



directed graph G

$$S_{ij} := (P_{ij},\ Q_{ij})$$

$$s_i := (p_i,\ q_i)$$

$$v_i := |V_i|^2, \quad \ell_{ij} := |I_{ij}|^2$$

Branch flow model

Branch flow model

linear

$$\sum_{j \to k} S_{jk} = \sum_{i \to j} \left(S_{ij} - z_{ij} \ell_{ij} \right) + s_j$$

$$v_i - v_j = 2 \ \mathrm{Re} \left(z_{ij}^H S_{ij} \right) - \left| z_{ij} \right|^2 \ell_{ij}$$

quadratic

$$v_i \ell_{ij} = \left| S_{ij} \right|^2$$

$$x := (s, v, S, \ell) \in \mathbf{R}^{3(m+n+1)}$$

$$= \ (p, q, v, P, Q, \ell)$$

DistFlow equations (radial nk)

Baran & Wu, 1989

Branch flow model

Bus injection model

$$s_j = \sum_{k: j \sim k} y_{jk}^H \left(|V_j|^2 - V_j V_k^H \right)$$

$$(V, s) \in \mathbf{C}^{2(n+1)}$$

Branch flow model

$$\sum_{j \to k} S_{jk} = \sum_{i \to j} \left(S_{ij} - z_{ij} \ell_{ij} \right) + s_j$$

$$v_i - v_j = 2 \, \mathrm{Re} \left(z_{ij}^H S_{ij} \right) - |z_{ij}|^2 \ell_{ij}$$

$$v_i \ell_{ij} = |S_{ij}|^2$$

$$x := (s, v, S, \boldsymbol{\ell}) \in \mathbf{R}^{3(m+n+1)}$$

DistFlow equations (radial nk)
Baran & Wu, 1989

Branch flow model

Bus injection model

$$s_j = \sum_{k: j\sim k} y_{jk}^H \left(|V_j|^2 - V_j V_k^H \right)$$

$$(V, s) \in \mathbf{C}^{2(n+1)}$$

Branch flow model

$$\sum_{j \to k} S_{jk} = \sum_{i \to j} \left(S_{ij} - z_{ij} \ell_{ij} \right) + s_j$$

$$v_i - v_j = 2 \,\mathrm{Re}\left(z_{ij}^H S_{ij} \right) - |z_{ij}|^2 \ell_{ij}$$

$$v_i \ell_{ij} = |S_{ij}|^2$$

\+ cycle condition on

$$x := (s, v, S, \ell) \in \mathbf{R}^{3(m+n+1)}$$

Cycle condition

A relaxed solution x satisfies the cycle condition if

$$\$q \quad \text{s.t.} \quad Bq = b(x) \qquad \mod 2p$$

incidence matrix;
depends on topology

$$x := (S, \ell, v, s)$$

$$b_{jk}(x) := \text{Ð}\left(v_j - z_{jk}^H S_{jk}\right)$$

Branch flow model

Bus injection model

$$s_j = \sum_{k:j\sim k} y_{jk}^H \left(|V_j|^2 - V_j V_k^H \right)$$

$$(V, s) \in \mathbf{C}^{2(n+1)}$$

Branch flow model

$$\sum_{j\to k} S_{jk} = \sum_{i\to j} \left(S_{ij} - z_{ij} \ell_{ij} \right) + s_j$$

$$v_i - v_j = 2\ \mathrm{Re}\left(z_{ij}^H S_{ij} \right) - |z_{ij}|^2 \ell_{ij}$$

$$v_i \ell_{ij} = |S_{ij}|^2$$

+ cycle condition on

$$x := (s, v, S, \ell) \in \mathbf{R}^{3(m+n+1)}$$

<u>Theorem</u>: BIM = BFM

[Farivar & Low 2013 TPS
Bose et al 2012 Allerton]

Branch flow model

- BFM and BIM are **equivalent** (nonlinear bijection)
- … but some results are easier to formulate or prove in one than the other
- BFM is much more numerically stable
- BFM is useful for radial networks
  - Extremely efficient computation (BFS)
  - Much better linearization
  - Compact extension to multiphase unbalanced nk

Gan & L PSCC 2014

Branch flow model

Bus injection model

$$s_j = \sum_{k: j\sim k} y_{jk}^H \left(|V_j|^2 - V_j V_k^H \right)$$

$$(V, s) \in \mathbf{C}^{2(n+1)}$$

Branch flow model

$$\sum_{j\to k} S_{jk} = \sum_{i\to j} \left(S_{ij} - z_{ij}\ell_{ij} \right) + s_j$$

$$v_i - v_j = 2\ \mathrm{Re}\left(z_{ij}^H S_{ij} \right) - |z_{ij}|^2 \ell_{ij}$$

$$v_i \ell_{ij} = |S_{ij}|^2$$

+ cycle condition on

$$x := (s, v, S, \ell) \in \mathbf{R}^{3(m+n+1)}$$

SOCP relaxation

Bus injection model

$$s_j = \sum_{k: j \sim k} y_{jk}^H \left(|V_j|^2 - V_j V_k^H \right)$$

$$(V, s) \in \mathbf{C}^{2(n+1)}$$

Branch flow model

$$\sum_{j \to k} S_{jk} = \sum_{i \to j} \left(S_{ij} - z_{ij} \ell_{ij} \right) + s_j$$

$$v_i - v_j = 2 \, \mathrm{Re} \left(z_{ij}^H S_{ij} \right) - |z_{ij}|^2 \ell_{ij}$$

$$v_i \ell_{ij} \geq |S_{ij}|^2$$

$$x := (s, v, S, \ell) \in \mathbf{R}^{3(m+n+1)}$$

SOCP relaxation of OPF

OPF: $$\min_{x \in \mathbf{X}} f(x)$$

SOCP: $$\min_{x \in \mathbf{X}^+} f(x)$$

SOCP relaxation of OPF

OPF: $\min_{x \in \mathbf{X}} f(x)$

SOCP: $\min_{x \in \mathbf{X}^+} f(x)$

But all these algorithms are offline …

… unsuitable for real-time optimization of network of distributed energy resources

SOCP relaxation of OPF

OPF: $\min_{x \in \mathbf{X}} f(x)$

SOCP: $\min_{x \in \mathbf{X}^+} f(x)$

We will compare our online algorithm to SOCP relaxation wrt optimality and speed

OPF

$$\min \quad \sum_{i=0}^{n} a_i p_i^2 + b_i p_i$$

$$\text{over} \quad x := (p_i, q_i, i \in N)$$

controllable devices

$$y := (p_0, q_0, v_i, i \in N; P_{ij}, Q_{ij}, \ell_{ij}, (i,j) \in E)$$

$$\text{s.t.}$$

uncontrollable state

OPF

$$\min \quad \sum_{i=0}^{n} a_i p_i^2 + b_i p_i$$

$$\text{over} \quad x := (p_i, q_i, i \in N)$$ controllable devices

$$y := (p_0, q_0, v_i, i \in N; P_{ij}, Q_{ij}, \ell_{ij}, (i,j) \in E)$$

$$\text{s.t.} \quad F(x, y) = 0$$ BFM (DistFlow, radial network)

$$\underline{v}_i \leq v_i \leq \overline{v}_i, \qquad i \in N$$

$$x \in X := \{\underline{x} \leq x \leq \overline{x}\}$$

$$\text{Assume:} \quad \frac{\partial F}{\partial y} \neq 0 \qquad \Rightarrow \qquad y(x) \quad \text{over } X$$

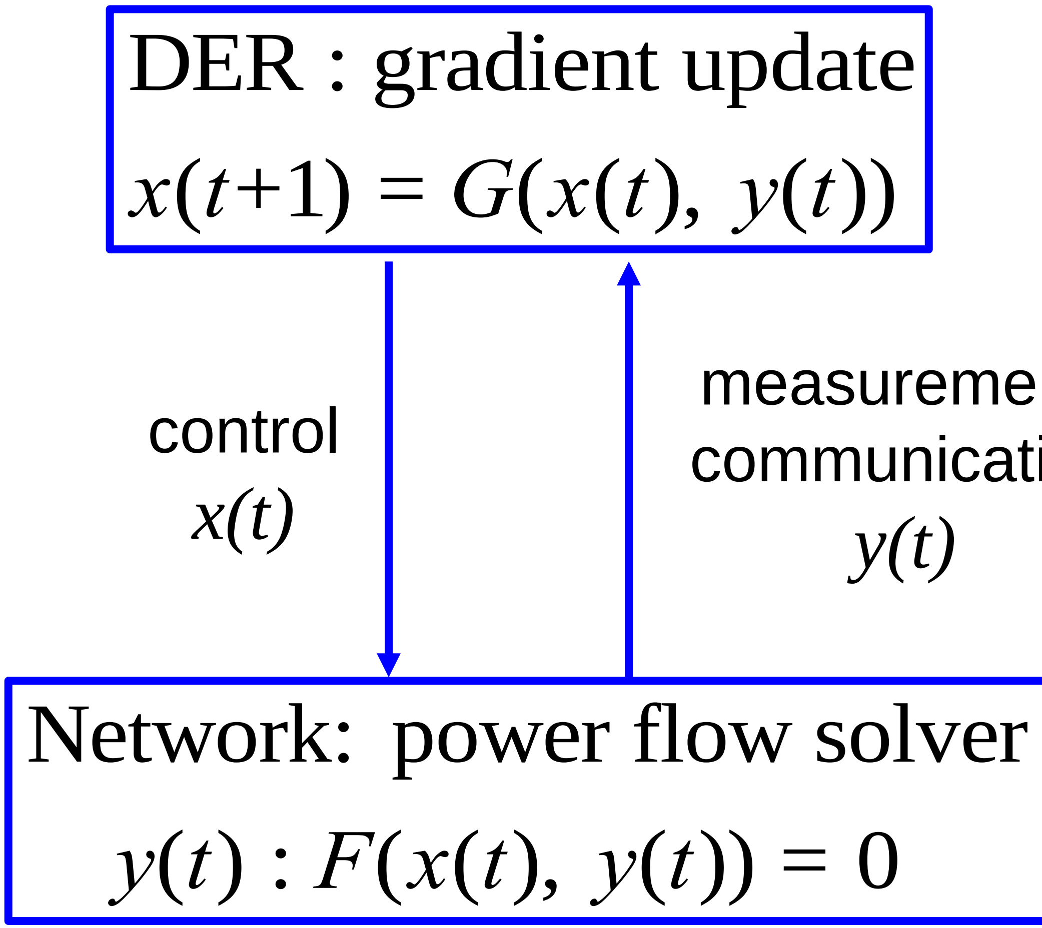
Eliminate y from OPF

$$\min \quad a_0 p_0^2(x) + b_0 p_0(x) + \sum_{i=1}^{n} (a_i p_i^2 + b_i p_i)$$

$$\text{over} \quad x \in X := \{ \underline{x} \le x \le \overline{x} \}$$

$$\text{s.t.} \quad \underline{v}_i \le v_i(x) \le \overline{v}_i, \qquad i \in N$$

Online (real-time) perspective



Bolognani et al arXiv 2013
Gan & Low JSAC 2016
Dall'Anese & Simonetto 2016

Approximate OPF

$$\min \quad a_0 p_0^2(x) + b_0 p_0(x) + \sum_{i=1}^{n} (a_i p_i^2 + b_i p_i)$$

$$\text{over} \quad x \in X := \{ \underline{x} \le x \le \overline{x} \}$$

$$\text{s.t.} \quad \underline{v}_i \le v_i(x) \le \overline{v}_i, \qquad i \in N$$

add log barrier function
to objective to remove
voltage constraints

$$\min \quad L(x, y(x);\ \mu)$$

$$\text{over} \quad x \in X := \{ \underline{x} \le x \le \overline{x} \}$$

L: nonconvex

Approximate OPF

$$\begin{aligned} \min \quad & L(x, y(x);\ \mu) \\ \text{over} \quad & x \in X := \{ \underline{x} \le x \le \overline{x} \} \end{aligned}$$

Recap: OPF → approximate OPF

- Reduce to x only by eliminating y using power flow equations
- Add barrier function on $v(x)$ to remove voltage constraints

Online gradient algorithm

$$\begin{aligned} \min \quad & L(x, y(x);\ \mu) \\ \text{over} \quad & x \in X := \{\underline{x} \le x \le \overline{x}\} \end{aligned}$$

gradient projection algorithm:

$$x(t+1) = \left[x(t) - \eta \frac{\partial L}{\partial x}(t) \right]_X \qquad \text{active control}$$

$$y(t) = y(x(t)) \qquad \text{law of physics}$$

- Explicitly exploits network to carry out part of algorithm
- Algorithm naturally tracks changing network conditions

Online gradient algorithm

$$
\begin{array}{ll}
\min & L(x, y(x);\ \mu) \\
\text{over} & x \in X := \{\underline{x} \le x \le \overline{x}\}
\end{array}
$$

gradient projection algorithm:

$$x(t+1) = \left[x(t) - \eta \frac{\partial L}{\partial x}(t) \right]_X \qquad \text{active control}$$

$$y(t) = y(x(t)) \qquad \text{law of physics}$$

Results

1. Local optimality
2. Global optimality
3. Suboptimality bound

[Gan & Low JSAC 2016]

Local optimality

- $x(t)$ converges to set of local optima
- if #local optima is finite, $x(t)$ converges

Global optimality

Assume: $p_0(x)$ convex over X

$v_k(x)$ concave over X

$$A := \{ x \in X \; : \; v(x) \le a_k \overline{v} + b_k \underline{v} \}$$

Theorem

If all local optima are in A then

- $x(t)$ converges to the set of global optima
- $x(t)$ itself converges a global optimum if #local optima is finite

Global optimality

Assume: $p_0(x)$ convex over X

$v_k(x)$ concave over X

$$A := \{x \in X \; : \; v(x) \le a_k \overline{v} + b_k \underline{v}\}$$

Theorem

- Can choose $\left(a_k, b_k\right)$ s.t.

  $A \rightarrow$ original feasible set

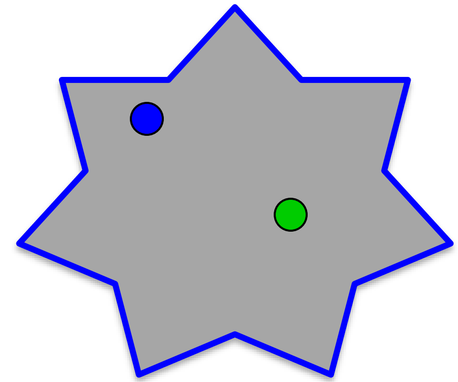
- If SOCP is exact over X, then assumption holds

Suboptimality gap

any local optimum

any original feasible pt slightly away from boundary

$$L(x^*) - L(\hat{x}) \leq r \approx 0$$



- Informally, a local minimum is almost as good as any strictly interior feasible point

Simulations

# bus	CVX		IPM		error	speedup
	obj	time(s)	obj	time(s)		
42	10.4585	6.5267	10.4585	0.2679	-0.0e-7	24.36
56	34.8989	7.1077	34.8989	0.3924	+0.2e-7	18.11
111	0.0751	11.3793	0.0751	0.8529	+5.4e-6	13.34
190	0.1394	20.2745	0.1394	1.9968	+3.3e-6	10.15
290	0.2817	23.8817	0.2817	4.3564	+1.1e-7	5.48
390	0.4292	29.8620	0.4292	2.9405	+5.4e-7	10.16
490	0.5526	36.3591	0.5526	3.0072	+2.9e-7	12.09
590	0.7035	43.6932	0.7035	4.4655	+2.4e-7	9.78
690	0.8546	51.9830	0.8546	3.2247	+0.7e-7	16.12
790	0.9975	62.3654	0.9975	2.6228	+0.7e-7	23.78
890	1.1685	67.7256	1.1685	2.0507	+0.8e-7	33.03
990	1.3930	74.8522	1.3930	2.7747	+1.0e-7	26.98
1091	1.5869	83.2236	1.5869	1.0869	+1.2e-7	76.57
1190	1.8123	92.4484	1.8123	1.2121	+1.4e-7	76.27
1290	2.0134	101.0380	2.0134	1.3525	+1.6e-7	74.70
1390	2.2007	111.0839	2.2007	1.4883	+1.7e-7	74.64
1490	2.4523	122.1819	2.4523	1.6372	+1.9e-7	74.83
1590	2.6477	157.8238	2.6477	1.8021	+2.0e-7	87.58
1690	2.8441	147.6862	2.8441	1.9166	+2.1e-7	77.06
1790	3.0495	152.6081	3.0495	2.0603	+2.1e-7	74.07
1890	3.8555	160.4689	3.8555	2.1963	+1.9e-7	73.06
1990	4.1424	171.8137	4.1424	2.3586	+1.9e-7	72.84

Outline

Optimal power flow

- DistFlow model and ACOPF
- Online algorithm
- Analysis and simulations

Gan & L, JSAC 2016

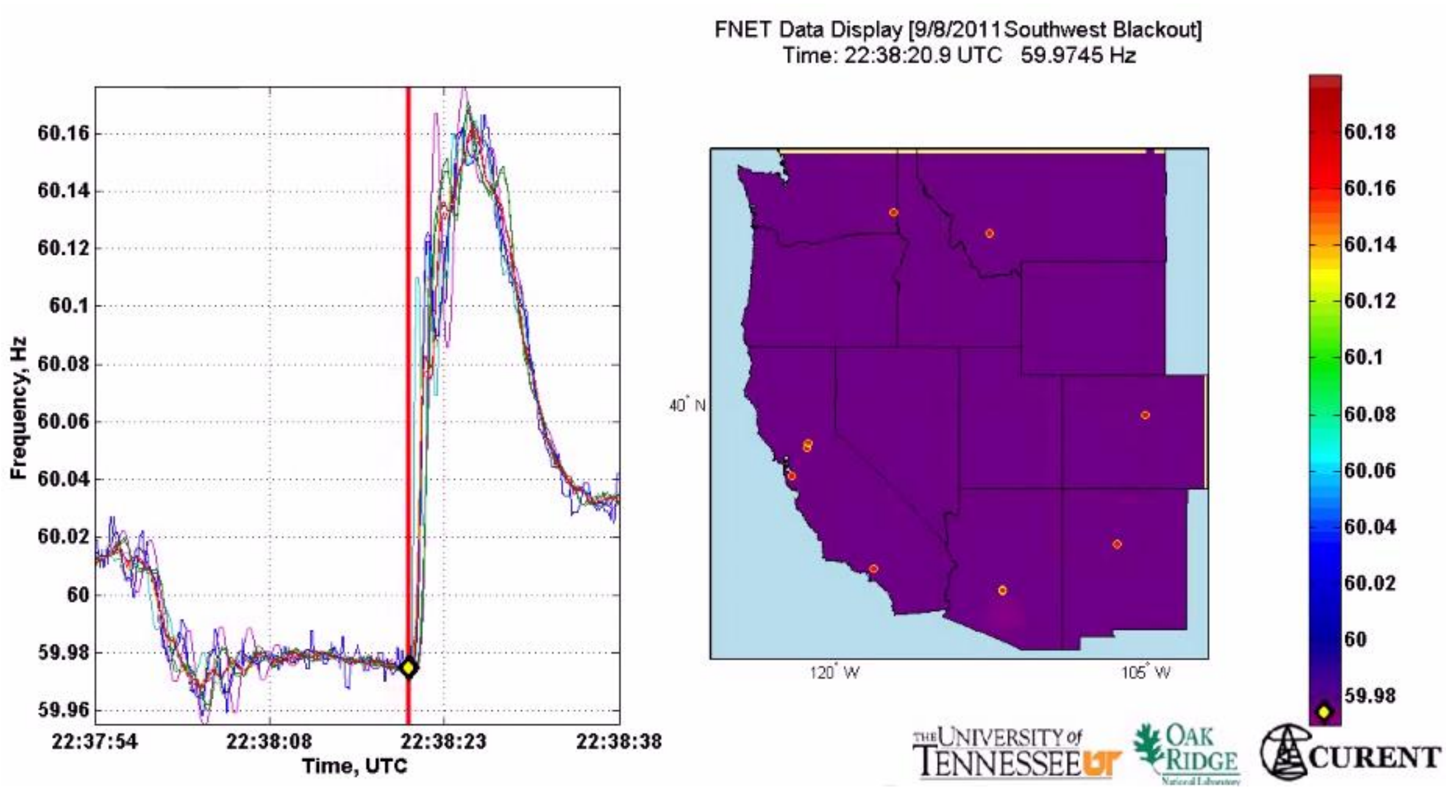
Load-side frequency control

- Dynamic model & design approach
- Distributed online algorithm
- Analysis and simulations
- Details

Zhao, Topcu, Li, L, TAC 2014
Mallada, Zhao, L, Allerton 2014
Zhao et al: CDC 2014, CISS 2015, PSCC 2016

Motivation

- All buses synchronized to same nominal frequency (US: 60 Hz; Europe/China: 50 Hz)
- Supply-demand imbalance ➔ frequency fluctuation



in)

2011 Southwest blackout

How

How to design load-side frequency control ?

How does it interact with generator-side control ?

Literature: load-side control

Original idea & early analytical work

- Schweppe et al 1980; Bergin, Hill, Qu, Dorsey, Wang, Varaiya …

Small scale trials around the world

- D.Hammerstrom et al 2007, UK Market Transform Programme 2008

Early simulation studies

- Trudnowski et al 2006, Lu and Hammerstrom 2006, Short et al 2007, Donnelly et al 2010, Brooks et al 2010, Callaway and I. A. Hiskens, 2011, Molina-Garcia et al 2011

Analytical work – load-side control

- Zhao et al (2012/2014), Mallada and Low (2014), Mallada et al (2014), Zhao and Low (2014), Zhao et al (2015)
- Simpson-Porco et al 2013, You and Chen 2014, Zhang and Papachristodoulou (2014), Ma et al (2014), Zhao, et al (2014),

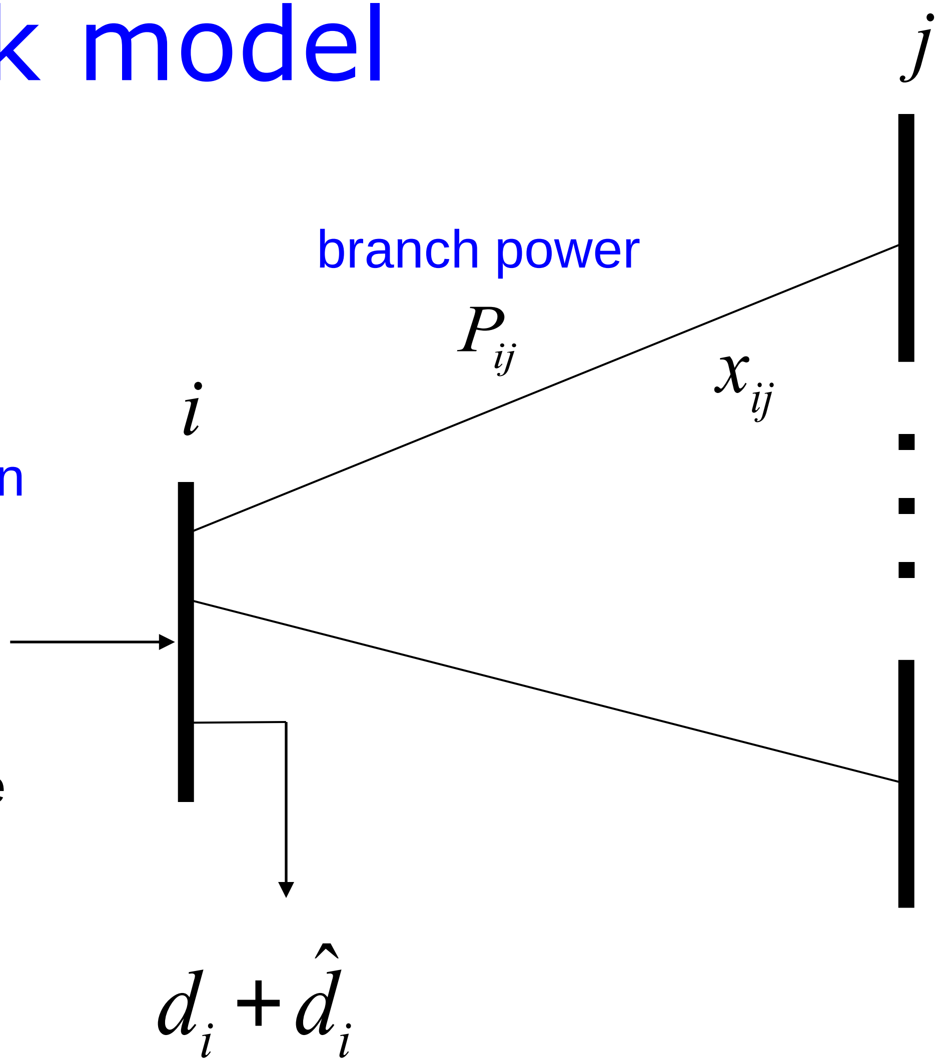
Recent analysis – generator-side/microgrid control:

- Andreasson et al (2013), Zhang and Papachristodoulou (2013), Li et al (2014), Burger et al (2014), You and Chen (2014), Simpson-Porco et al (2013), Hill et al (2014), Dorfler et al (2014)

Network model

generation

P_i^m



Will include generator-side control later

loads:
controllable + freq-sensitive

i : region/control area/balancing authority

Network model

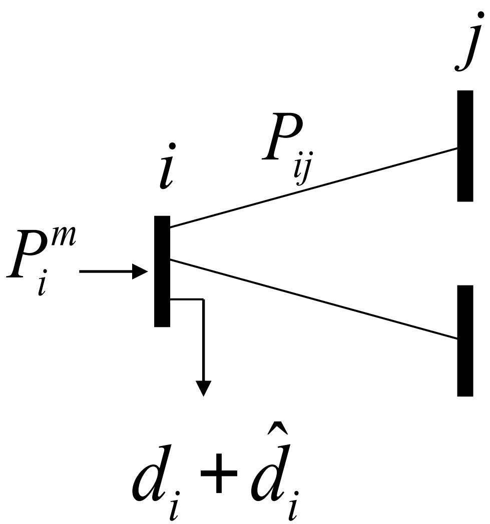
$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

Generator bus: $M_i > 0$

Load bus: $M_i = 0$

Damping/uncontr loads: $\hat{d}_i = D_i \omega_i$

Controllable loads: d_i

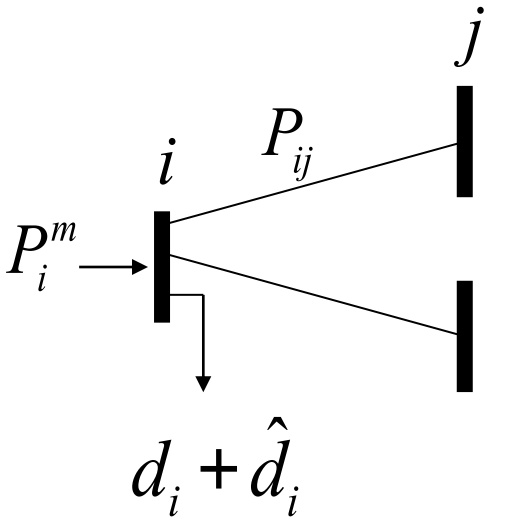


Network model

$$M_i \dot{w}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(w_i - w_j\right) \qquad \forall \; i \to j$$

- swing dynamics
- all variables are deviations from nominal
- extends to nonlinear power flow



Frequency control

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall\ i \rightarrow j$$

Suppose the system is in steady state

$$\dot{\omega}_i = 0 \qquad \dot{P}_{ij} = 0 \qquad \omega_i = 0$$

Then: disturbance in gen/load …

Frequency control

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall \; i \to j$$

current approach

load-side control

Outline

Network model

Distributed online algorithm

Simulations

Details

Main references (frequency control):

Zhao, Topcu, Li, L, TAC 2014
Mallada, Zhao, L, Allerton 2014
Zhao et al: CDC 2014, CISS 2015, PSCC 2016

Load-side controller design

$$M_i \dot{w}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(w_i - w_j\right) \qquad \forall\ i \to j$$

Control goals

Zhao, Topcu, Li, Low
TAC 2014
Mallada, Zhao, Low
Allerton, 2014

- Rebalance power & stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows
- Respect line limits

Load-side controller design

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right) \qquad \forall\ i \rightarrow j$$

Control goals (while min disutility)

Zhao, Topcu, Li, Low
TAC 2014

Mallada, Zhao, Low
Allerton, 2014

- Rebalance power & stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows
- Respect line limits

Load-side controller design

Design control law whose equilibrium solves:

$$
\begin{aligned}
\min_{d,P} \quad & \sum_i c_i(d_i) && && \text{load disutility} \\
\text{s. t.} \quad & P_i^m - d_i = \sum_e C_{ie} P_e && \text{node } i && \text{power balance} \\
& \sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k && \text{area } k && \text{inter-area flows} \\
& \underline{P}_e \le P_e \le \overline{P}_e && \text{line } e && \text{line limits}
\end{aligned}
$$

Control goals (while min disutility)

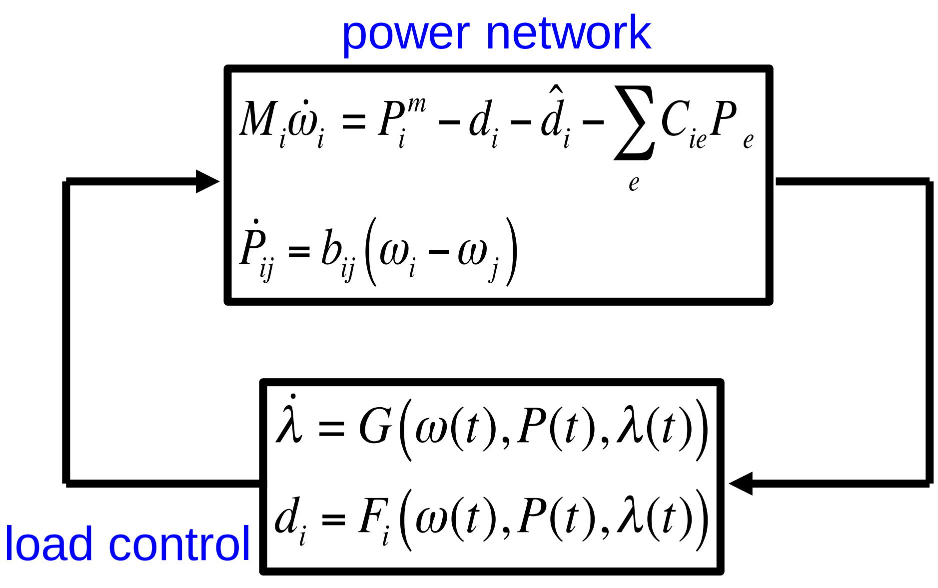
- Rebalance power & stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows
- Respect line limits

freq will emerge as Lagrange multiplier for power imbalance

Load-side controller design

Design control (G, F) s.t. closed-loop system

- is stable
- has equilibrium that is optimal



$$\min_{d,P} \quad \sum_i c_i(d_i)$$

$$\text{s. t.} \quad P_i^m - d_i = \sum_e C_{ie} P_e \qquad \text{node } i$$

$$\sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k \qquad \text{area } k$$

$$\underline{P}_e \le P_e \le \overline{P}_e \qquad \text{line } e$$

Load-side controller design

Idea: exploit system dynamic as part of primal-dual algorithm for modified opt

- Distributed algorithm
- Stability analysis
- Control goals in equilibrium

power network

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right)$$

load control

$$\dot{\lambda} = G\left(\omega(t), P(t), \lambda(t)\right)$$

$$d_i = F_i\left(\omega(t), P(t), \lambda(t)\right)$$

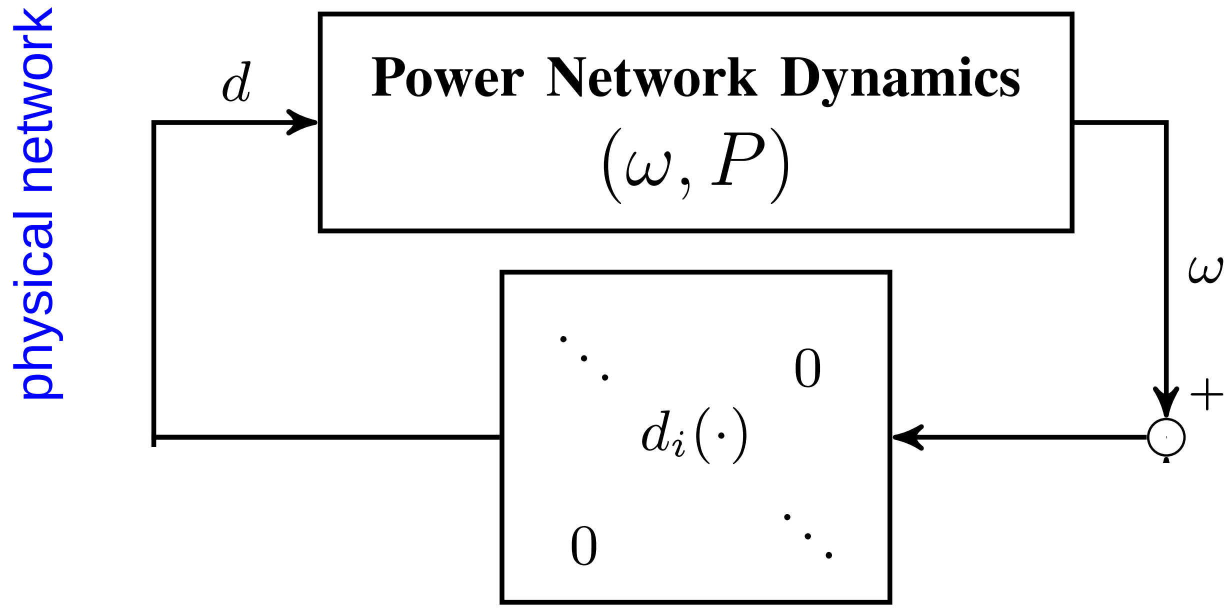
$$\min_{d,P} \quad \sum_i c_i(d_i)$$

$$\text{s. t.} \quad P_i^m - d_i = \sum_e C_{ie} P_e \qquad \text{node } i$$

$$\sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k \qquad \text{area } k$$

$$\underline{P}_e \le P_e \le \overline{P}_e \qquad \text{line } e$$

Summary: control architecture

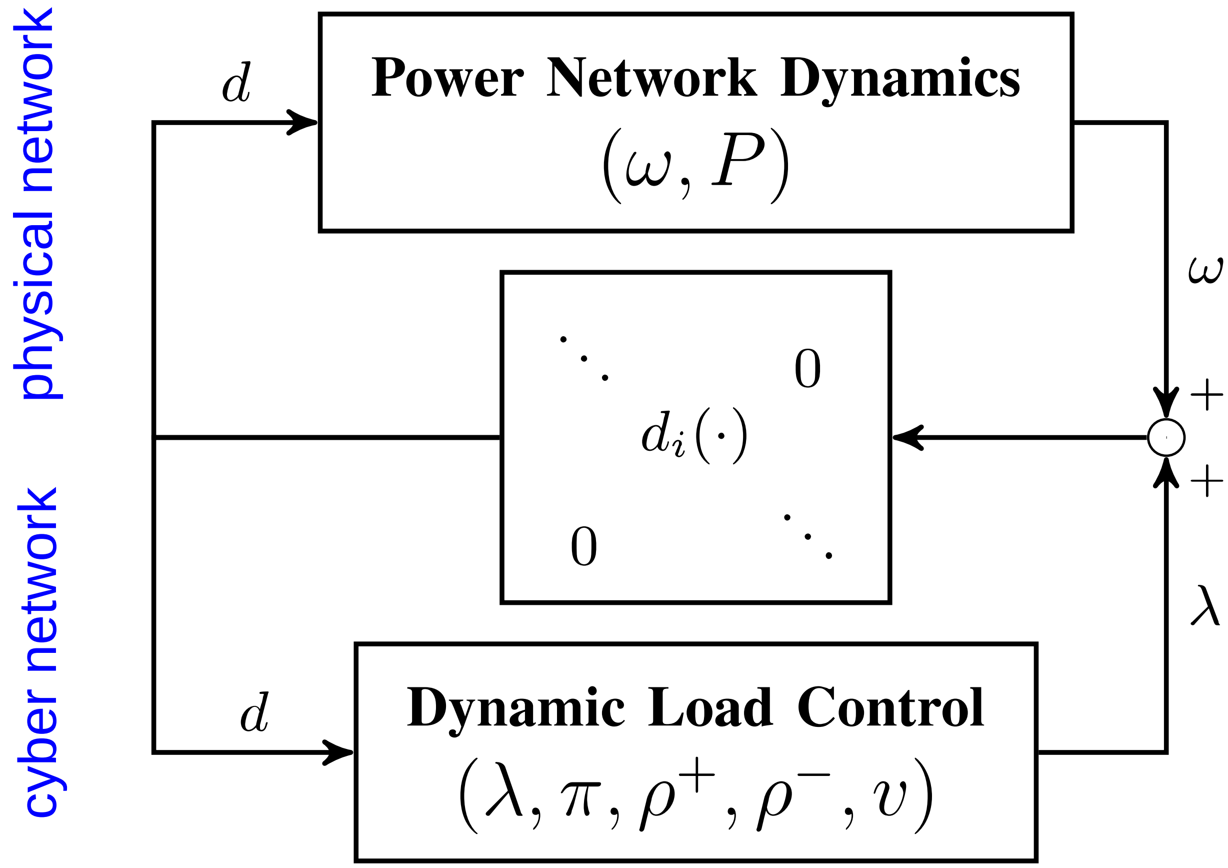


Primary load-side frequency control

- completely decentralized
- Theorem: stable dynamic, optimal equilibrium

Zhao, Topcu, Li, Low. TAC 2014

Summary: control architecture

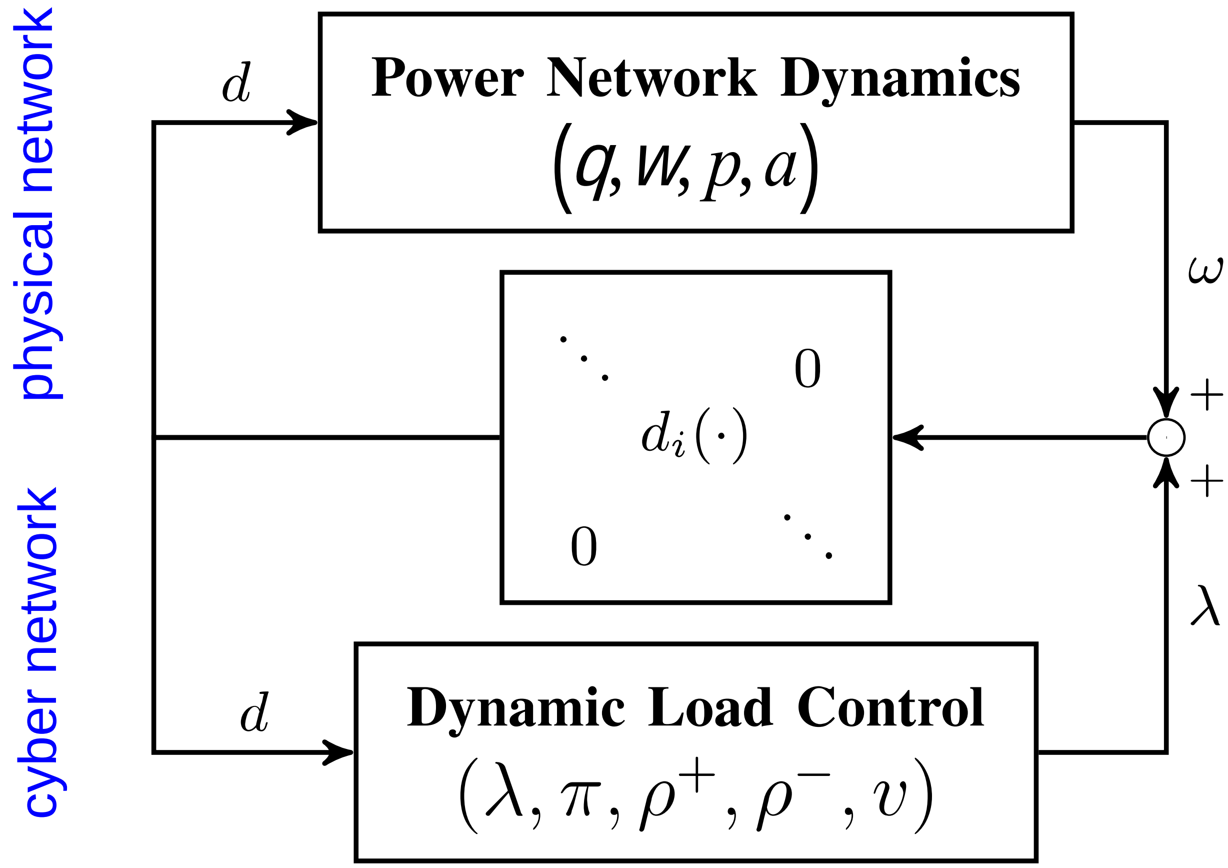


Secondary load-side frequency control

- communication with neighbors
- Theorem: stable dynamic, optimal equilibrium

Mallada, Zhao, Low. Allerton 2014

Summary: control architecture



With **generator-side** control, **nonlinear** power flow

- load-side improves both transient & eq
- <u>Theorem</u>: stable dynamic, optimal equilibrium

Zhao, Mallada, Low. CISS 2015

Outline

Network model

Load-side frequency control

Simulations

Details

Main references (frequency control):

Zhao, Topcu, Li, L, TAC 2014
Mallada, Zhao, L, Allerton 2014
Zhao et al: CDC 2014, CISS 2015, PSCC 2016

Simulations

Dynamic simulation of IEEE 39-bus system

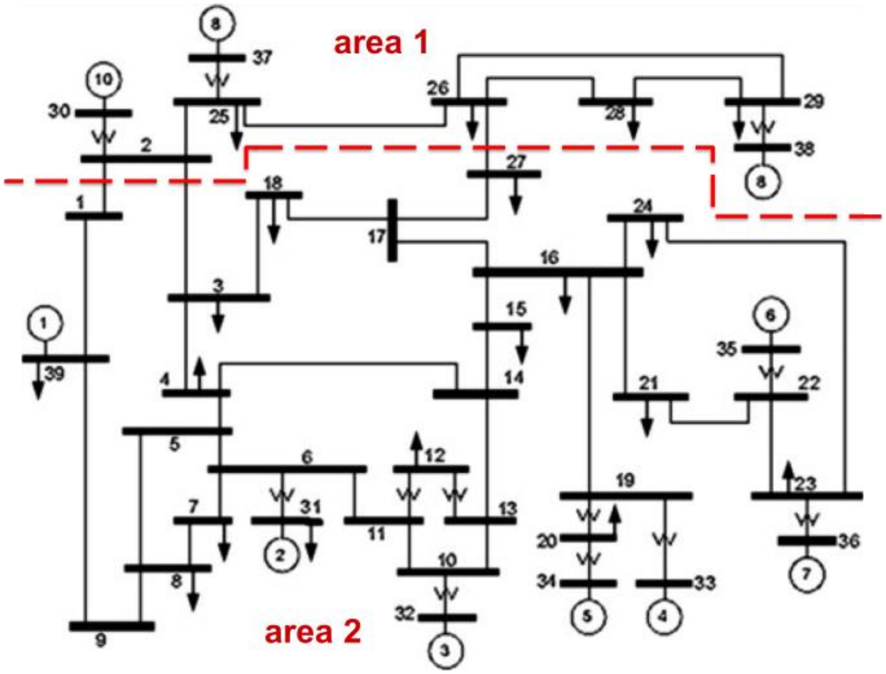
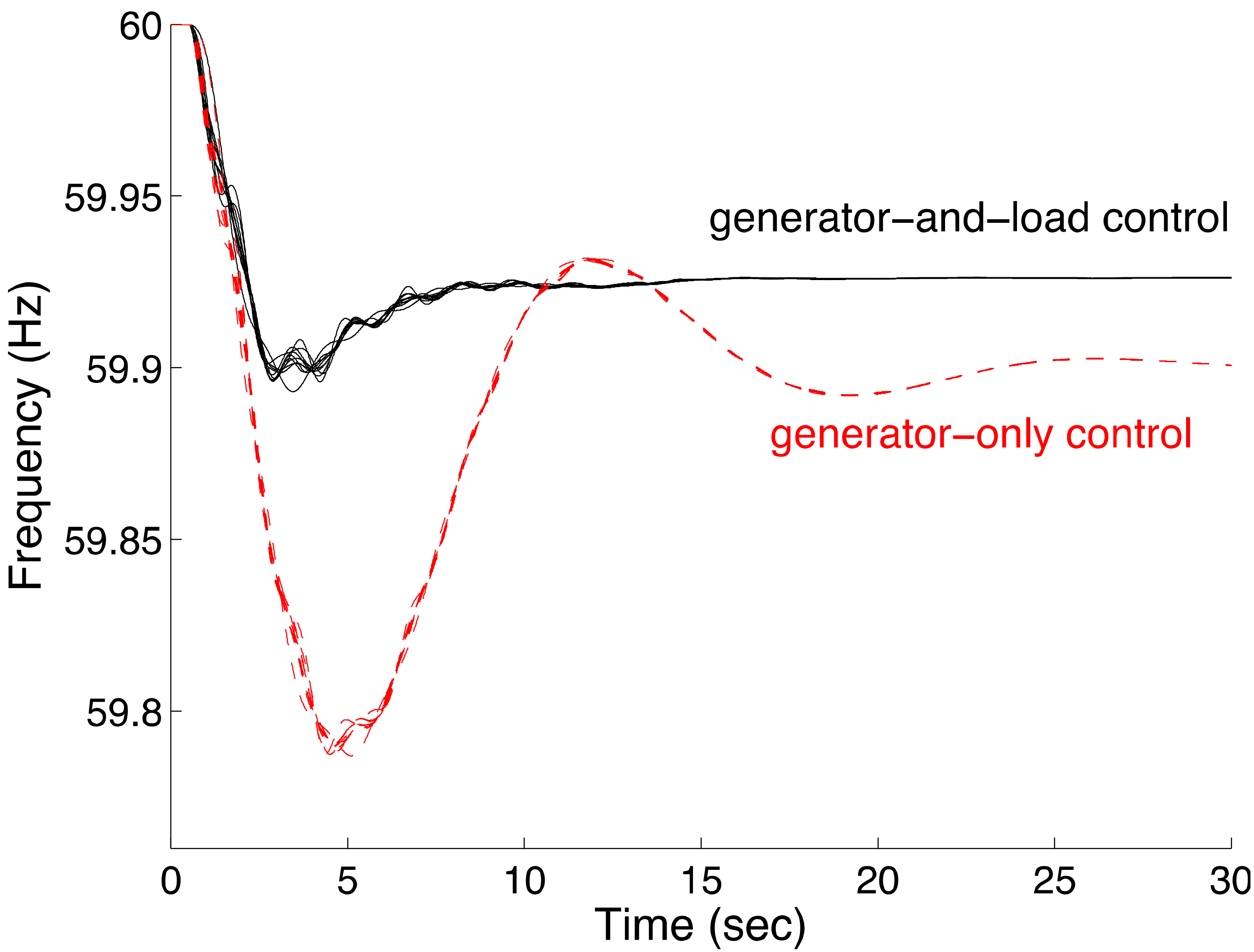


Fig. 2: IEEE 39 bus system : New England

- Power System Toolbox (RPI)
- Detailed generation model
- Exciter model, power system stabilizer model
- Nonzero resistance lines

Primary control



Secondary control

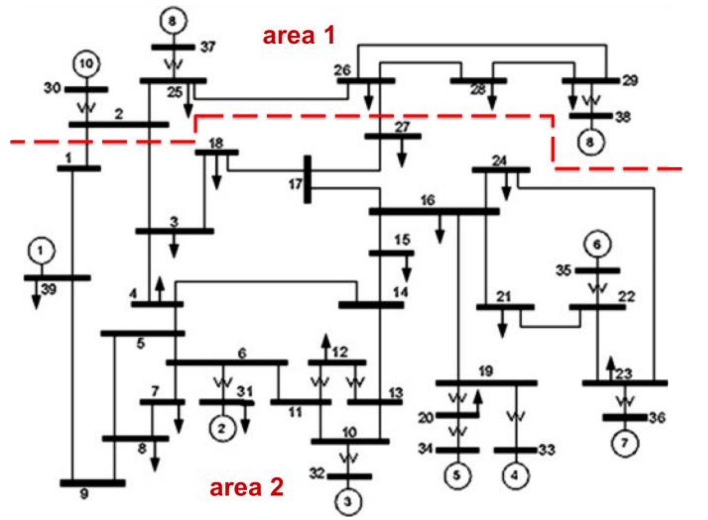
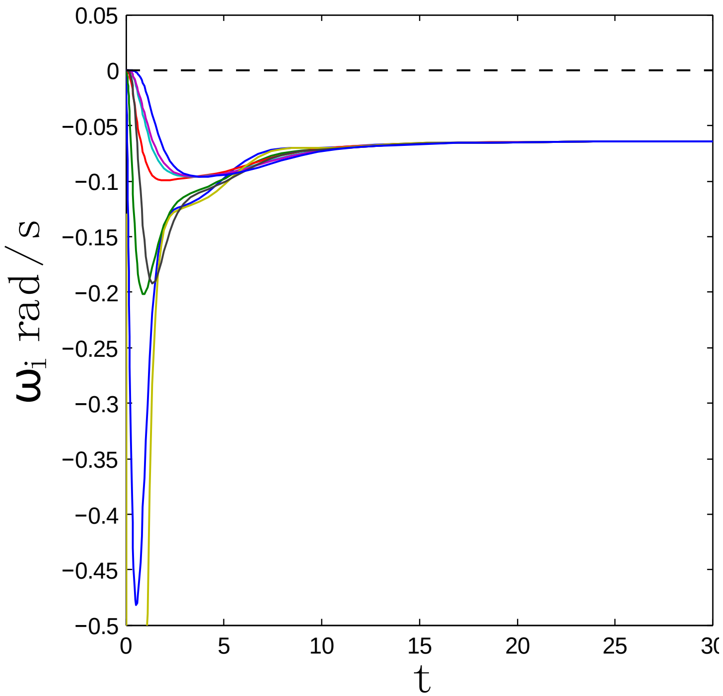
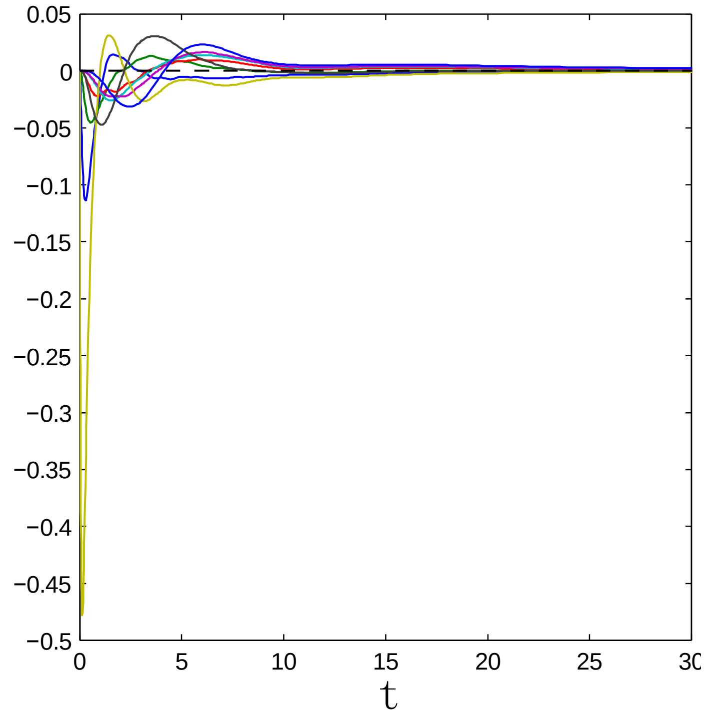


Fig. 2: IEEE 39 bus system: New England

swing dynamics



with OLC



area 1

Secondary control

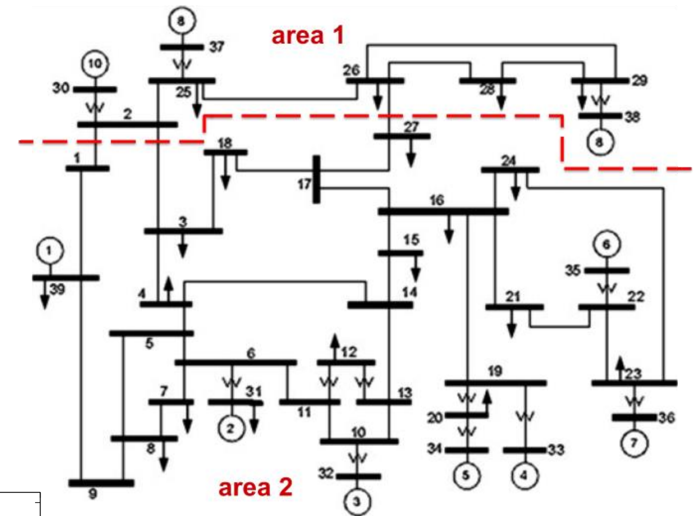
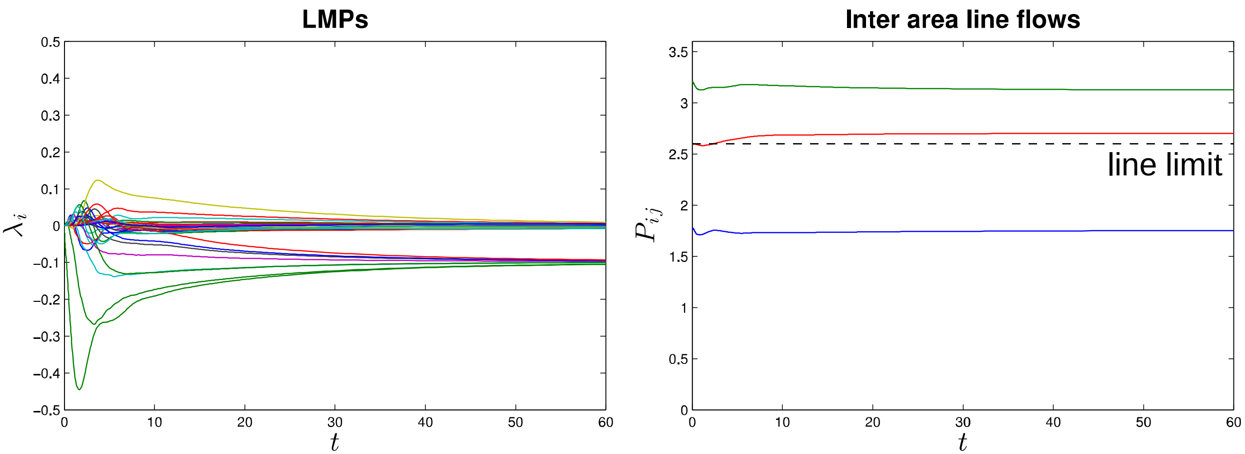
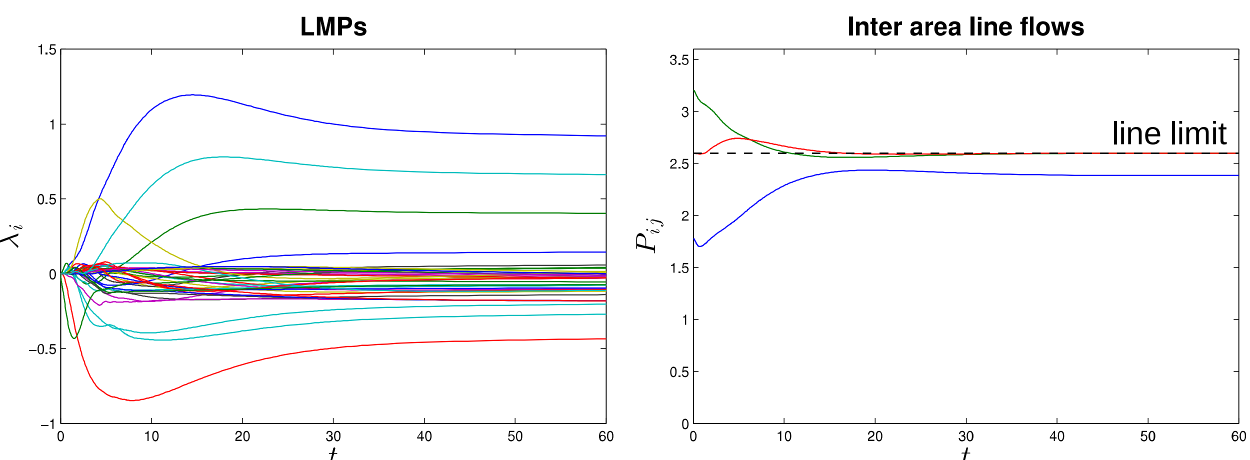


Fig. 2: IEEE 39 bus system: New England



no line limits

Total inter-area flow is the same in both cases



with line limits

Key message

Large network of DERs

- Real-time optimization at scale
- Computational challenge: power flow solution

Online optimization (feedback control)

- Network solves hard problem in real time for free
- Exploit it for our optimization/control
- Naturally adapts to evolving network conditions

Examples

- Slow timescale: OPF
- Fast timescale: frequency control

more details

(backup)

Recall: design approach

Idea: exploit system dynamic as part of primal-dual algorithm for modified opt

- closed-loop system is stable
- its equilibria are optimal

power network

$$M_i \dot{\omega}_i = P_i^m - d_i - \hat{d}_i - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i - \omega_j\right)$$

$$\dot{\lambda} = G\big(\omega(t), P(t), \lambda(t)\big)$$

$$d_i = F_i\big(\omega(t), P(t), \lambda(t)\big)$$

load control

$$\min_{d,P} \quad \sum_i c_i(d_i)$$

s. t.

$$P_i^m - d_i = \sum_e C_{ie} P_e \qquad \text{node } i$$

$$\sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k \qquad \text{area } k$$

$$\underline{P}_e \le P_e \le \overline{P}_e \qquad \text{line } e$$

Outline

Load-side frequency control

- Primary control   Zhao et al SGC2012, Zhao et al TAC2014
- Secondary control
- Interaction with generator-side control

Optimal load control (OLC)

$$\min_{d,\hat{d},P} \quad \sum_i \left(c_i(d_i) + \frac{\hat{d}_i^2}{2D_i} \right)$$

$$\text{s. t.} \quad P_i^m - \left(d_i + \hat{d}_i \right) = \sum_e C_{ie} P_{ie} \quad \forall i$$

demand = supply

disturbances

controllable loads

$$\min_{d,P} \quad \sum_i c_i(d_i)$$

$$\text{s. t.} \quad P_i^m - d_i = \sum_e C_{ie} P_e \qquad \text{node } i$$

$$\sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k \qquad \text{area } k$$

$$\underline{P}_e \le P_e \le \overline{P}_e \qquad \text{line } e$$

system dynamics + load control = primal dual alg

swing dynamics

$$\dot{\omega}_i = -\frac{1}{M_i}\left(d_i(t) + D_i \omega_i(t) - P_i^m + \sum_{i \to j} P_{ij}(t) - \sum_{j \to i} P_{ji}(t) \right)$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i(t) - \omega_j(t)\right)$$

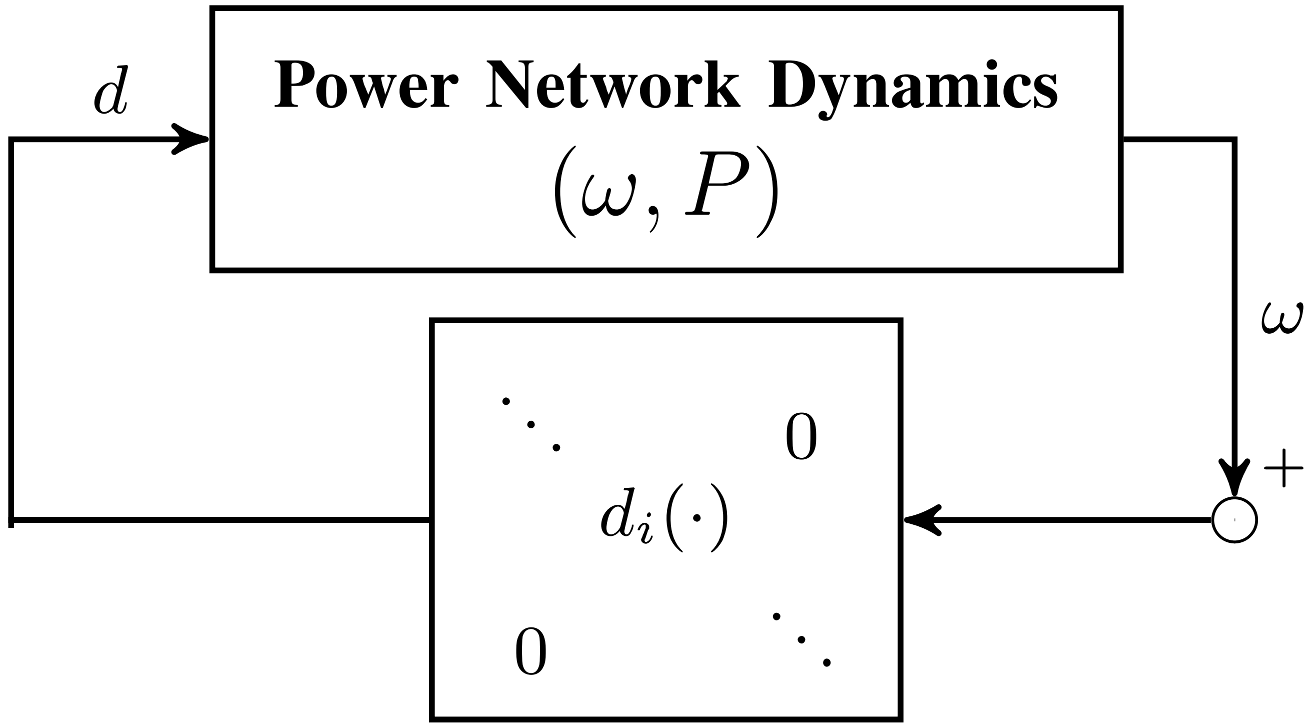
implicit

load control

$$d_i(t) := \left[c_i'^{-1}\left(\omega_i(t)\right) \right]_{\underline{d}_i}^{\bar{d}_i}$$

active control

Control architecture



Load-side primary control works

Theorem

Starting from any $\left(d(0),\ \hat{d}(0),\ w(0),\ P(0)\right)$

system trajectory $\left(d(t),\ \hat{d}(t),\ w(t),\ P(t)\right)$

converges to $\left(d^*,\ \hat{d}^*,\ w^*,\ P^*\right)$ as $t \to \infty$

- $\left(d^*,\ \hat{d}^*\right)$ is unique optimal of OLC
- w^* is unique optimal for dual

- completely decentralized
- frequency deviations contain right info for local decisions that are globally optimal

Recap: control goals

Yes
- Rebalance power

Yes
- Stabilize frequencies

No
- Restore nominal frequency $\left(W^{*}\ 1\ 0\right)$

No
- Restore scheduled inter-area flows

No
- Respect line limits

Outline

Load-side frequency control

- Primary control
- Secondary control
- Interaction with generator-side control

Mallada, Low, IFAC 2014
Mallada et al, Allerton 2014

OLC for secondary control

$$\min_{d,\hat{d},P,v} \quad \sum_i \left(c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 \right)$$

s. t.

$$P^m - (d + \hat{d}) = CP \qquad \text{demand = supply}$$

$$P^m - d = CBC^T v \qquad \text{restore nominal freq}$$

$$\min_{d,P} \quad \sum_i c_i(d_i)$$

s. t.

$$P_i^m - d_i = \sum_e C_{ie}P_e \qquad \text{node } i$$

$$\sum_{i \in N_k} \sum_e C_{ie}P_e = \hat{P}_k \qquad \text{area } k$$

$$\underline{P}_e \le P_e \le \overline{P}_e \qquad \text{line } e$$

OLC for secondary control

$$\min_{d,\hat{d},P,v} \quad \sum_i \left(c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 \right)$$

$$\text{s. t.} \quad P^m - (d + \hat{d}) = CP \qquad \text{demand = supply}$$

$$P^m - d = CBC^T v \qquad \text{restore nominal freq}$$

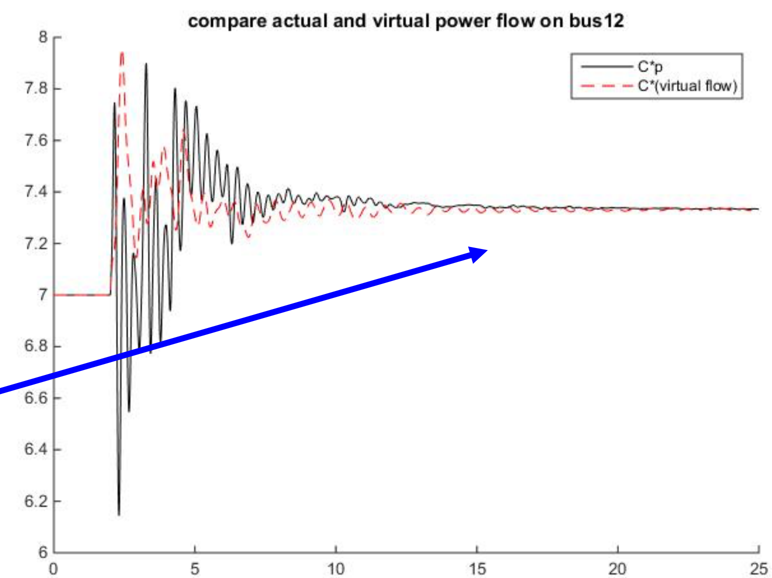
key idea: "virtual flows"

$$BC^T v$$

in steady state:

virtual flow = real flows

$$BC^T v = P$$



OLC for secondary control

$$\min_{d,\hat{d},P,v} \quad \sum_i \left(c_i(d_i) + \frac{1}{2D_i}\hat{d}_i^2 \right)$$

s. t.

$$P^m - (d + \hat{d}) = CP \qquad \text{demand = supply}$$

$$P^m - d = CBC^T v \qquad \text{restore nominal freq}$$

$$\hat{C}BC^T v = \hat{P} \qquad \text{restore inter-area flow}$$

$$\underline{P} \le BC^T v \le \overline{P} \qquad \text{respect line limit}$$

in steady state:

virtual flow = real flows

$$BC^T v = P$$

Recall: primary control

swing dynamics:

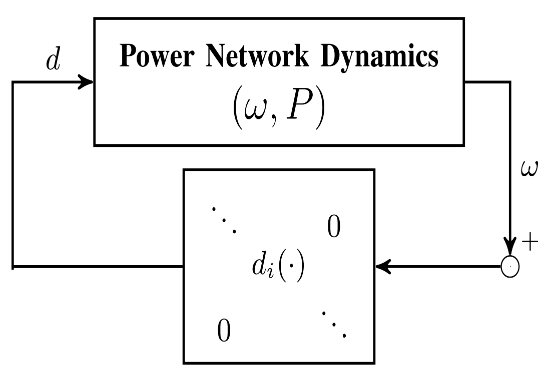
$$\dot{\omega}_i = -\frac{1}{M_i}\left(d_i(t) + D_i\omega_i(t) - P_i^m + \sum_{e\in E} C_{ie}P_e(t)\right)$$

$$\dot{P}_{ij} = b_{ij}\left(\omega_i(t) - \omega_j(t)\right)$$

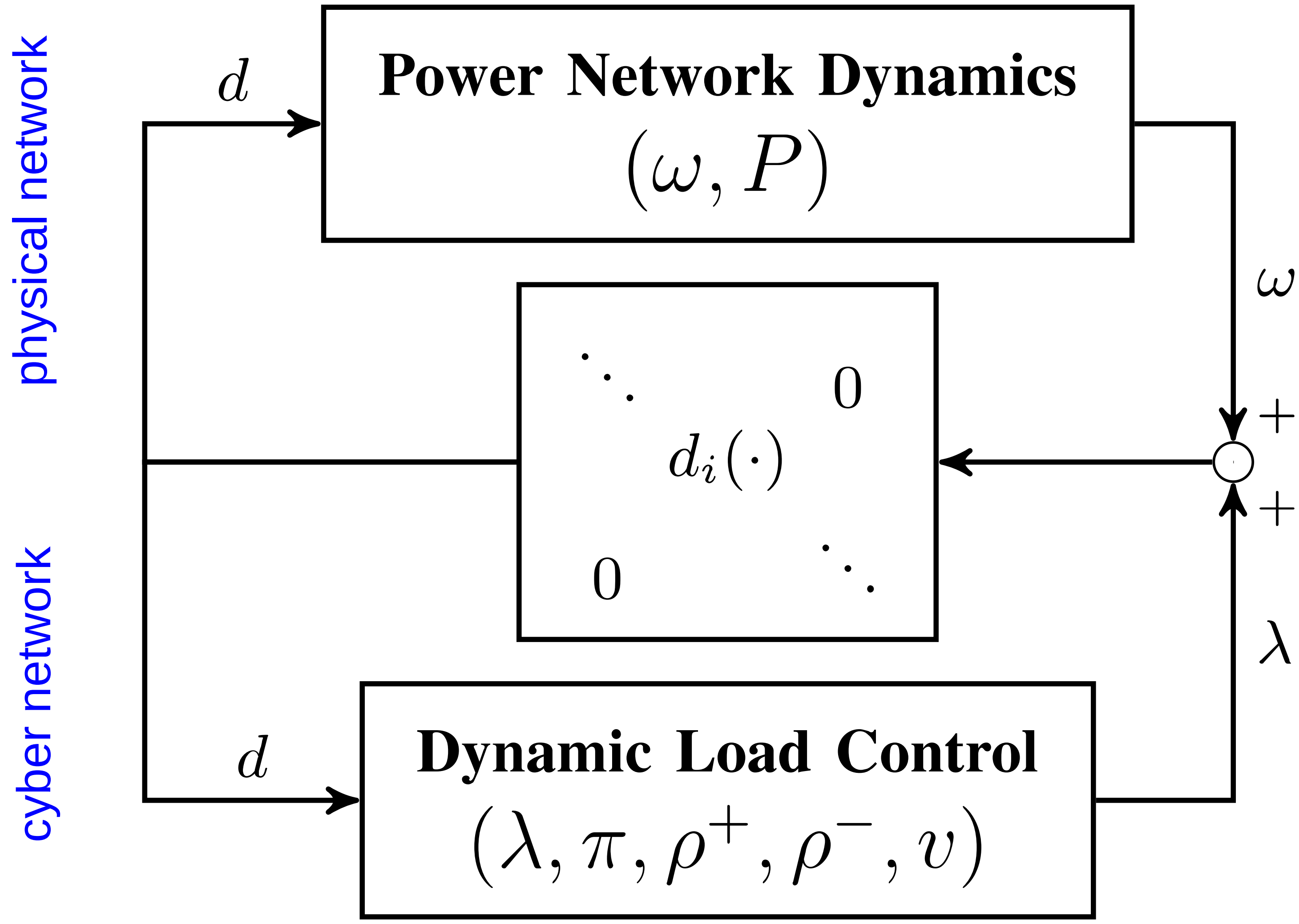
implicit

load control: $$d_i(t) := \left[c_i'^{-1}\left(\omega_i(t)\right)\right]_{\underline{d}_i}^{\overline{d}_i}$$

active control



Control architecture



Secondary frequency control

load control: $d_i(t) := \left[c_i'^{-1}\left(\omega_i(t) + \lambda_i(t)\right) \right]_{\underline{d}_i}^{\bar{d}_i}$

computation & communication:

primal var: $$\dot{v} = \chi^v \left(L_B \lambda - C D_B \hat{C}^T \pi - C D_B (\rho^+ - \rho^-) \right)$$

dual vars: $$\dot{\lambda} = \zeta^\lambda \left(P^m - d - L_B v \right)$$

$$\dot{\pi} = \zeta^\pi \left(\hat{C} D_B C^T v - \hat{P} \right)$$

$$\dot{\rho}^+ = \zeta^{\rho^+} \left[D_B C^T v - \bar{P} \right]_{\rho^+}^+$$

$$\dot{\rho}^- = \zeta^{\rho^-} \left[\underline{P} - D_B C^T v \right]_{\rho^-}^+$$

Secondary control works

<u>**Theorem**</u>

starting from any initial point, system trajectory converges s. t.

- $\left(d^*, \hat{d}^*, P^*, v^*\right)$ is unique optimal of OLC
- nominal frequency is restored $w^* = 0$
- inter-area flows are restored $\hat{C}P^* = \hat{P}$
- line limits are respected $\underline{P}$ £ P^* £ $\overline{P}$

Recap: key ideas

Design optimal load control (OLC) problem

- Objective function, constraints

Derive control law as primal-dual algorithms

- Lyapunov stability
- Achieve original control goals in equilibrium

Distributed algorithms

primary control: $d_i(t) := c_i'^{-1}\left(\omega_i(t)\right)$

secondary control: $d_i(t) := c_i'^{-1}\left(\omega_i(t) + \lambda_i(t)\right)$

Recap: key ideas

Design optimal load control (OLC) problem

- Objective function, constraints

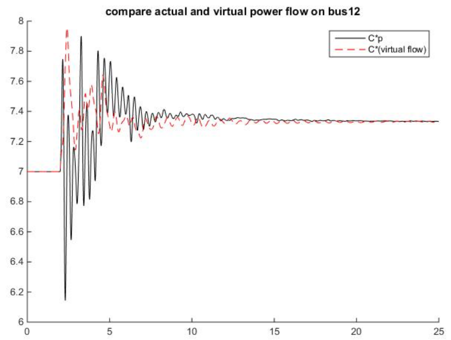
Derive control law as primal-dual algorithms

- Lyapunov stability
- Achieve original control goals in equilibrium

Distributed algorithms

Virtual flows

- Enforce desired properties on line flows



in steady state: virtual flow = real flows

$$BC^T v = P$$

Recap: control goals

- Yes — Rebalance power
- Yes — Resynchronize/stabilize frequency

Zhao, et al TAC2014

- Yes — Restore nominal frequency $\left(w^{*} \; 1 \; 0 \right)$
- Yes — Restore scheduled inter-area flows
- Yes — Respect line limits

Mallada, et al Allerton2014

Secondary control restores nominal frequency but requires local communication

Outline

Load-side frequency control

- Primary control
- Secondary control
- Interaction with generator-side control

Zhao and Low, CDC2014
Zhao, Mallada, Low, CISS 2015
Zhao, Mallada, Low, Bialek, PSCC 2016

Generator-side control

New model: nonlinear PF, with generator control

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + \boxed{p_i} - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \rightarrow j$$

Recall model: linearized PF, no generator control

$$M_i \dot{\omega}_i = -D_i \omega_i + \boxed{P_i^m - d_i} - \sum_e C_{ie} P_e$$

$$\dot{P}_{ij} = b_{ij} \left(\omega_i - \omega_j\right) \qquad \forall\ i \rightarrow j$$

Generator-side control

New model: nonlinear PF, with generator control

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + p_i - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \rightarrow j$$

generator bus: real power injection
load bus: controllable load

Generator-side control

New model: nonlinear PF, with generator control

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + p_i - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \rightarrow j$$

generator buses:

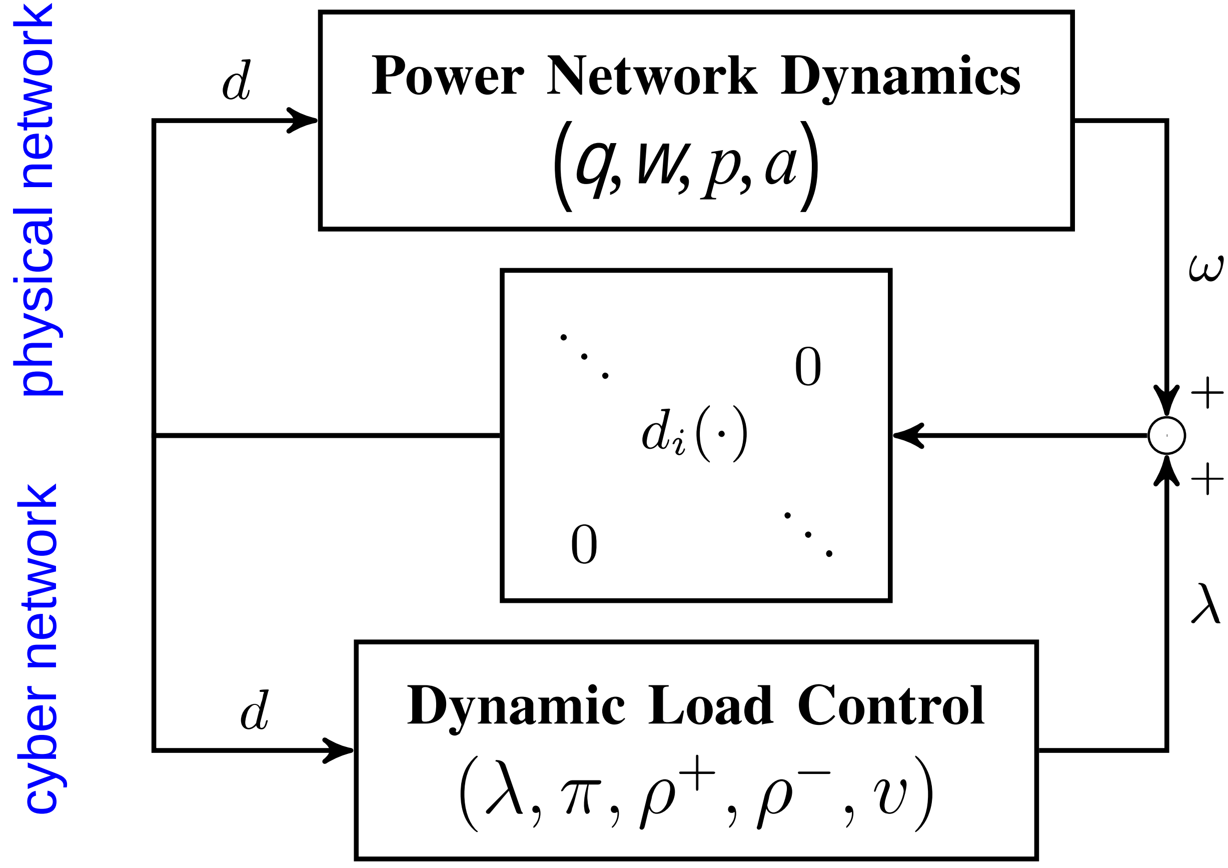
$$\dot{p}_i = -\frac{1}{\tau_{bi}}\left(p_i + a_i\right)$$

$$\dot{a}_i = -\frac{1}{\tau_{gi}}\left(a_i + p_i^c\right)$$

primary control $p_i^c(t) = p_i^c\left(w_i(t)\right)$

e.g. freq droop $p_i^c\left(w_i\right) = -\,b_i w_i$

Load-side control



Load-side primary control works

Theorem

- Every closed-loop equilibrium solves OLC and its dual

Suppose $\left|p_i^c(w) - p_i^c\left(w^*\right)\right| \pounds L_i \left|w - w^*\right|$

near w^* for some $L_i < D_i$

- Any closed-loop equilibrium is (locally) asymptotically stable provided

$$\left|q_i^* - q_j^*\right| < \frac{p}{2}$$

Conclusion

Forward-engineering design facilitates

- explicit control goals
- distributed algorithms
- stability analysis

Load-side frequency regulation

- primary & secondary control works
- helps generator-side control

Key message

Large network of DERs

- Real-time optimization at scale
- Computational challenge: power flow solution

Online optimization (feedback control)

- Network solves hard problem in real time for free
- Exploit it for our optimization/control
- Naturally adapts to evolving network conditions

Examples

- Slow timescale: OPF
- Fast timescale: frequency control