Optimal Power Flow:
online algorithm, fast dynamics

Steven Low

Caltech

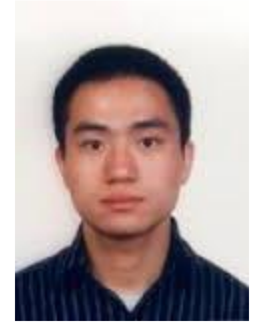
August 2016

OPF relaxation


Bose (UIUC)

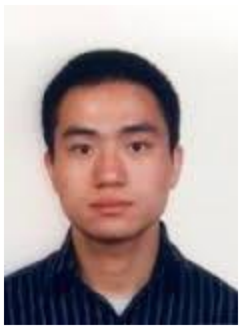

Chandy


Farivar

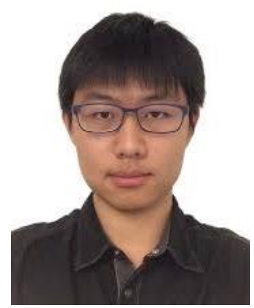

Gan (FB)


Lavaei (UCB)

Online OPF


Gan (FB)


Dvijotham (PNNL)


Tang

Dynamics

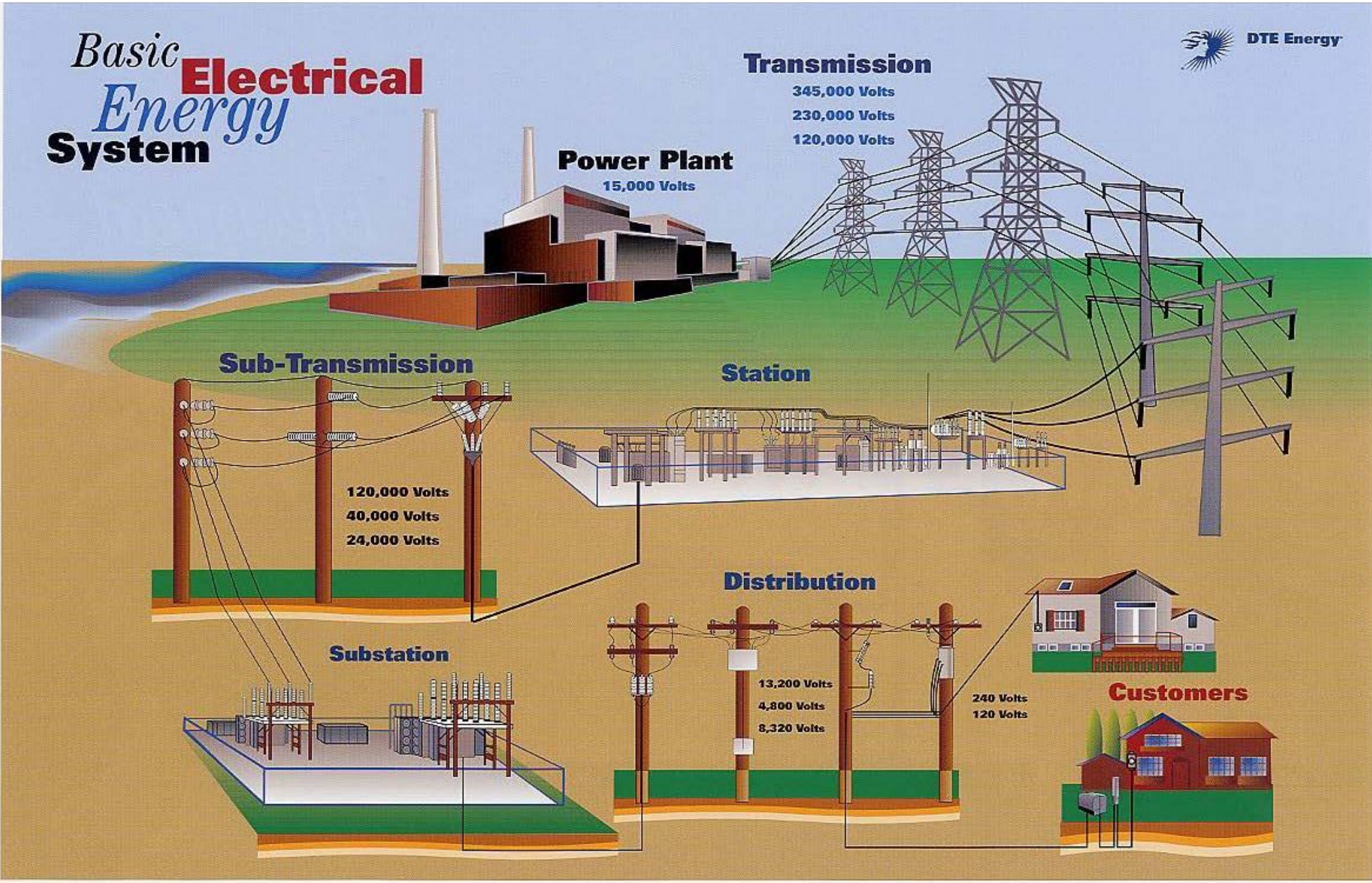

Bialek (Skoltech)


Li (Harvard)

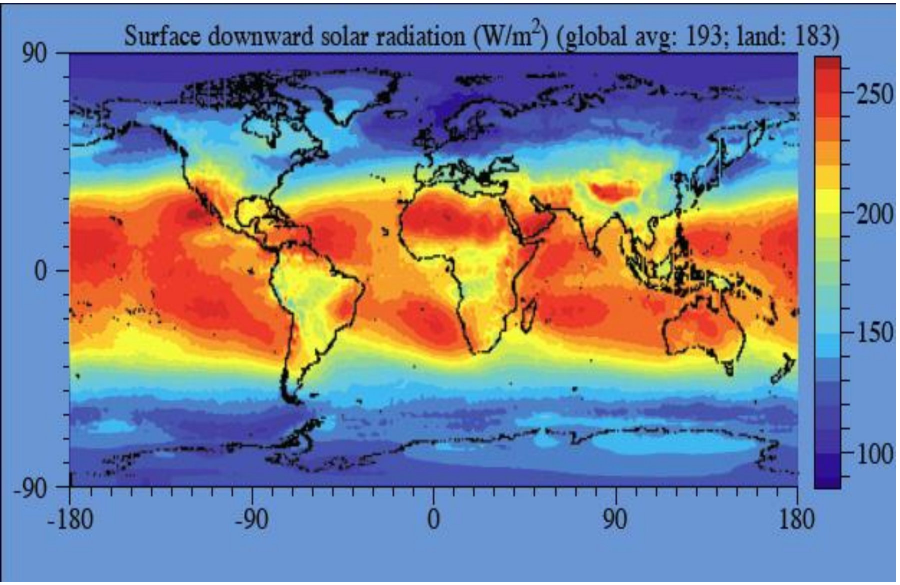

Mallada (JHU)

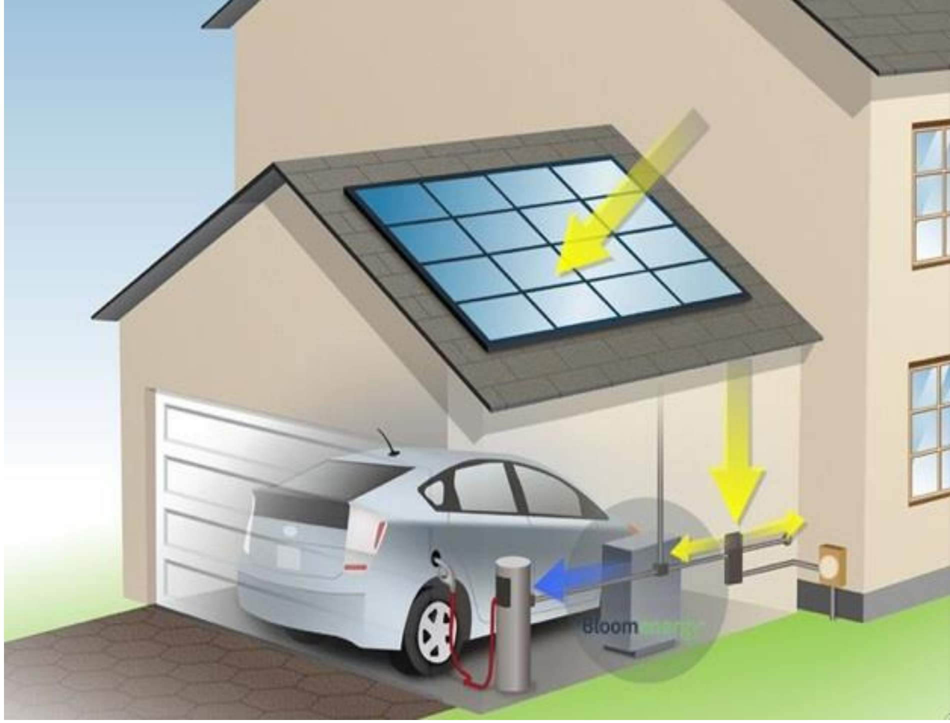

Topcu (Austin)


Zhao (NREL)


Basic Electrical Energy System
DTE Energy
Transmission
345,000 Volts
230,000 Volts
120,000 Volts
Power Plant
15,000 Volts
Sub-Transmission
Station
120,000 Volts
40,000 Volts
24,000 Volts
Distribution
Substation
13,200 Volts
4,800 Volts
8,320 Volts
240 Volts
120 Volts
Customers

Solar power over land:
> 20x world energy demand





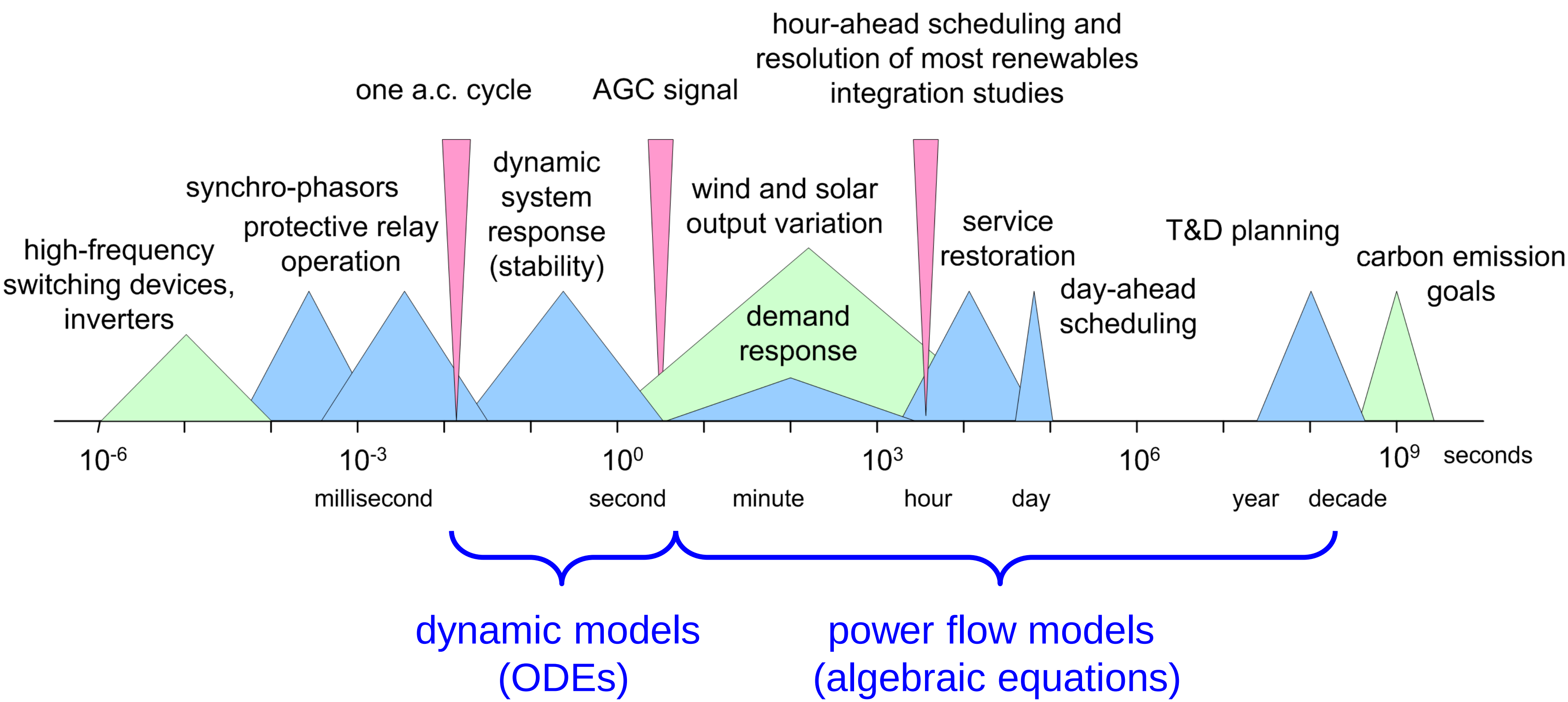
network of
billions of active
distributed energy
resources (DERs)

DER: PV, wind tb, EV, storage, smart bldg / appl

Multiple timescales

System dynamics and controls at different timescales

- require different models



Optimal power flow (OPF)

OPF is solved routinely for

- network control & optimization decisions
- market operations & pricing
- at timescales of mins, hours, days, …

Non-convex and hard to solve

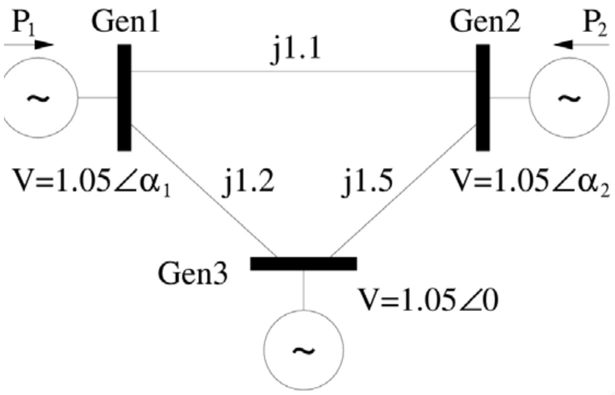
- Huge literature since 1962
- Common practice: DC power flow (LP)
- Also: Newton-Raphson, interior point, …

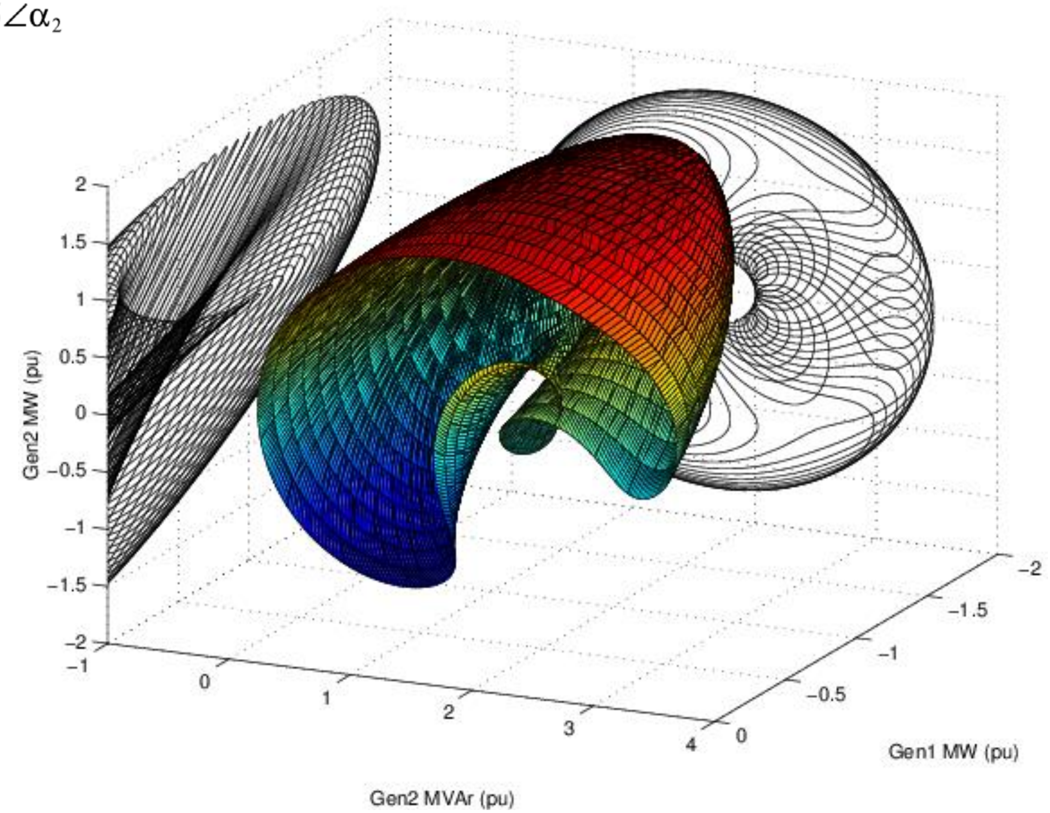
$$\min\ c(x) \quad \text{s. t.} \quad F(x) = 0,\ x \leq \overline{x}$$

Optimal power flow (OPF)

OPF problem underlies numerous applications

- nonlinearity of power flow equations ➔ nonconvexity





Ian Hiskens, Michigan

How to deal with nonconvexity of power flows?

Two ideas

1. exact semidefinite relaxation

Tutorial:
L, Convex relaxation of OPF, 2014
http://netlab.caltech.edu

How to deal with nonconvexity of power flows?

Two ideas

1. exact semidefinite relaxation

1. use grid as implicit power flow solver

Key message

Large network of DERs

- Real-time optimization at scale
- Computational challenge: power flow solution

Online optimization [feedback control]

- Network computes power flow solutions in real time at scale for free
- Exploit it for our optimization/control
- Naturally adapts to evolving network conditions

Examples

- Slow timescale: OPF
- Fast timescale: frequency control

Outline

Semidefinite relaxations of OPF

- Power flow models
- Offline algorithms

Tutorial:
L, Convex relaxation of OPF, 2014
http://netlab.caltech.edu

Online OPF

- Power flow models

Load-side frequency control

Zhao, Topcu, Li, L, TAC 2014

- Dynamic models

NSF · arpa·e · SOUTHERN CALIFORNIA EDISON · RESNICK INSTITUTE science + energy + sustainability · Skoltech Skolkovo Institute of Science and Technology

Online OPF

Gan (FB)

Dvijotham (PNNL)

Tang

Relaxations of OPF

OPF: $$\min_{x \in \mathbf{X}} f(x)$$

relaxation: $$\min_{\hat{x} \in \mathbf{X}^{+}} f(\hat{x})$$

But traditional algorithms are all offline …

… unsuitable for real-time optimization of
network of distributed energy resources

Relaxations of OPF

OPF: $$\min_{x \in \mathbb{X}} f(x)$$

relaxation: $$\min_{\hat{x} \in \mathbb{X}^+} f(\hat{x})$$

We will compare our online algorithm to relaxation wrt optimality and speed

OPF

$$\min \quad c_0(y) + c(x)$$

$$\text{over} \quad x, \; y$$

$$\text{s. t.}$$

controllable devices (x)

uncontrollable state (y)

OPF

$$\min \quad c_0(y) + c(x)$$

$$\text{over} \quad x,\ y$$

$$\text{s. t.} \quad F(x, y) = 0$$

power flow equations

OPF

$$\begin{aligned}
\min \quad & c_0(y) + c(x) \\
\text{over} \quad & x,\ y \\
\text{s. t.} \quad & F(x, y) = 0 && \text{power flow equations} \\
& y \le \overline{y} && \text{operational constraints} \\
& x \in X := \{ \underline{x} \le x \le \overline{x} \} && \text{capacity limits}
\end{aligned}$$

$$\text{Assume: } \frac{\partial F}{\partial y} \neq 0 \qquad \Rightarrow \qquad y(x) \ \text{ over } X$$

OPF: eliminate y

$$
\begin{aligned}
\min_{x} \quad & c_0(y(x)) + c(x) \\
\text{s. t.} \quad & y(x) \le \overline{y} \\
& x \in X := \{ \underline{x} \le x \le \overline{x} \}
\end{aligned}
$$

OPF: add barrier

$$\min_{x} \quad c_0(y(x)) + c(x)$$

$$\text{s. t.} \quad y(x) \le \overline{y}$$

$$x \in X := \{ \underline{x} \le x \le \overline{x} \}$$

add barrier function
to remove
operational constraints

$$\min \quad L(x, y(x);\ \mu)$$

$$\text{over} \quad x \in X$$

L: nonconvex

Online (feedback) perspective

DER : gradient update

$$x(t+1) = G(x(t),\ y(t))$$

cyber network

control $x(t)$

measurement, communication $y(t)$

Network: power flow solver

$$y(t) : F(x(t),\ y(t)) = 0$$

physical network

Online gradient algorithm

$$\min \quad L(x, y(x); \mu)$$
$$\text{over} \quad x \in X$$

gradient projection algorithm:

$$x(t+1) = \left[x(t) - \eta \frac{\partial L}{\partial x}(t) \right]_X \qquad \text{active control}$$

$$y(t) = y(x(t)) \qquad \text{law of physics}$$

- Explicitly exploits network as power flow solver
- Naturally tracks changing network conditions

Online gradient algorithm

$$\begin{array}{ll} \min & L(x, y(x);\ \mu) \\ \text{over} & x \in X \end{array}$$

gradient projection algorithm:

$$x(t+1) = \left[x(t) - \eta \frac{\partial L}{\partial x}(t) \right]_X \qquad \text{active control}$$

$$y(t) = y(x(t)) \qquad \text{law of physics}$$

Results

1. Optimality
2. Tracking performance

1. Local optimality

Under appropriate assumptions

- $x(t)$ converges to set of local optima
- if #local optima is finite, $x(t)$ converges

1. Global optimality

Assume: $p_0(x)$ convex over X

$v_k(x)$ concave over X

$$A := \{ x \in X \; : \; v(x) \le \alpha \bar{v} + (1-\alpha)\underline{v} \}$$

Theorem

If co{local optima} are in A then

- $x(t)$ converges to the set of global optima
- $x(t)$ itself converges a global optimum if #local optima is finite

1. Global optimality

Assume: $p_0(x)$ convex over X

$v_k(x)$ concave over X

$$A := \{ x \in X \ : \ v(x) \le \alpha \bar{v} + (1-\alpha)\underline{v} \}$$

Theorem

- Can choose α s.t.

  $A \rightarrow$ original feasible set

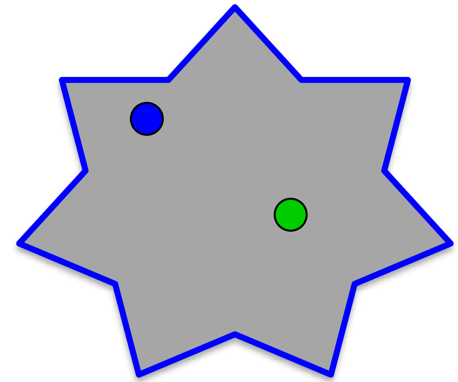
- If SOCP is exact over X, then assumption holds

1. Suboptimality gap

any local optimum

any original feasible pt slightly away from X boundary

$L(x^*) - L(\hat{x})$ £ r » 0



- Informally, a local minimum is almost as good as any strictly interior feasible point

2. Tracking performance

$$\min_{x} \quad c_0(y(x)) + c(x)$$

$$\text{s. t.} \quad y(x) \le \overline{y}$$

$$x \in X$$

static OPF

$$\min_{x} \quad c_0(y(x), \gamma_t) + c(x, \gamma_t)$$

$$\text{s. t.} \quad y(x, \gamma_t) \le \overline{y}$$

$$x \in X$$

drifting OPF

2. Tracking performance

$$R(x, x^*) := \sum_{t=1}^{T} c_0(y(x), g_t) + c(x, g_t)$$ cost of Alg

dynamic regret

$$- \sum_{t=1}^{T} c_0(y(x^*), g_t) + c(x^*, g_t)$$ optimal cost

2. Tracking performance

$$R(x, x^*) \;:=\; \sum_{t=1}^{T} c_0(y(x), \gamma_t) + c(x, \gamma_t)$$

cost of Alg

dynamic regret

$$-\; \sum_{t=1}^{T} c_0(y(x^*), \gamma_t) + c(x^*, \gamma_t)$$

optimal cost

Theorem

$$R(x, x^*) = O\left(\sqrt{T}\left(1 + \sum_{t=1}^{T} \left\| x^*_{t+1} - x^*_t \right\|\right)\right) + \sum_{t=1}^{T} \delta_t$$

rate of drifting

subopt of local min

2. Tracking performance

$$R(x, x^*) \;:=\; \sum_{t=1}^{T} c_0(y(x), g_t) + c(x, g_t)$$

cost of Alg

dynamic regret

$$- \sum_{t=1}^{T} c_0(y(x^*), g_t) + c(x^*, g_t)$$

optimal cost

<u>Theorem</u>

- If rate of drifting is $o\left(\sqrt{T}\right)$ then per-step $R(x, x^*)$ is asymptotically bounded by $\bar{\delta}$ (local min)

- Can made $\bar{\delta}$ arbitrarily small at cost of computation

Simulations

# bus	CVX		IPM		error	speedup
	obj	time(s)	obj	time(s)		
42	10.4585	6.5267	10.4585	0.2679	-0.0e-7	24.36
56	34.8989	7.1077	34.8989	0.3924	+0.2e-7	18.11
111	0.0751	11.3793	0.0751	0.8529	+5.4e-6	13.34
190	0.1394	20.2745	0.1394	1.9968	+3.3e-6	10.15
290	0.2817	23.8817	0.2817	4.3564	+1.1e-7	5.48
390	0.4292	29.8620	0.4292	2.9405	+5.4e-7	10.16
490	0.5526	36.3591	0.5526	3.0072	+2.9e-7	12.09
590	0.7035	43.6932	0.7035	4.4655	+2.4e-7	9.78
690	0.8546	51.9830	0.8546	3.2247	+0.7e-7	16.12
790	0.9975	62.3654	0.9975	2.6228	+0.7e-7	23.78
890	1.1685	67.7256	1.1685	2.0507	+0.8e-7	33.03
990	1.3930	74.8522	1.3930	2.7747	+1.0e-7	26.98
1091	1.5869	83.2236	1.5869	1.0869	+1.2e-7	76.57
1190	1.8123	92.4484	1.8123	1.2121	+1.4e-7	76.27
1290	2.0134	101.0380	2.0134	1.3525	+1.6e-7	74.70
1390	2.2007	111.0839	2.2007	1.4883	+1.7e-7	74.64
1490	2.4523	122.1819	2.4523	1.6372	+1.9e-7	74.83
1590	2.6477	157.8238	2.6477	1.8021	+2.0e-7	87.58
1690	2.8441	147.6862	2.8441	1.9166	+2.1e-7	77.06
1790	3.0495	152.6081	3.0495	2.0603	+2.1e-7	74.07
1890	3.8555	160.4689	3.8555	2.1963	+1.9e-7	73.06
1990	4.1424	171.8137	4.1424	2.3586	+1.9e-7	72.84

frequency control


Bialek (Skoltech)


Li (Harvard)


Mallada (JHU)

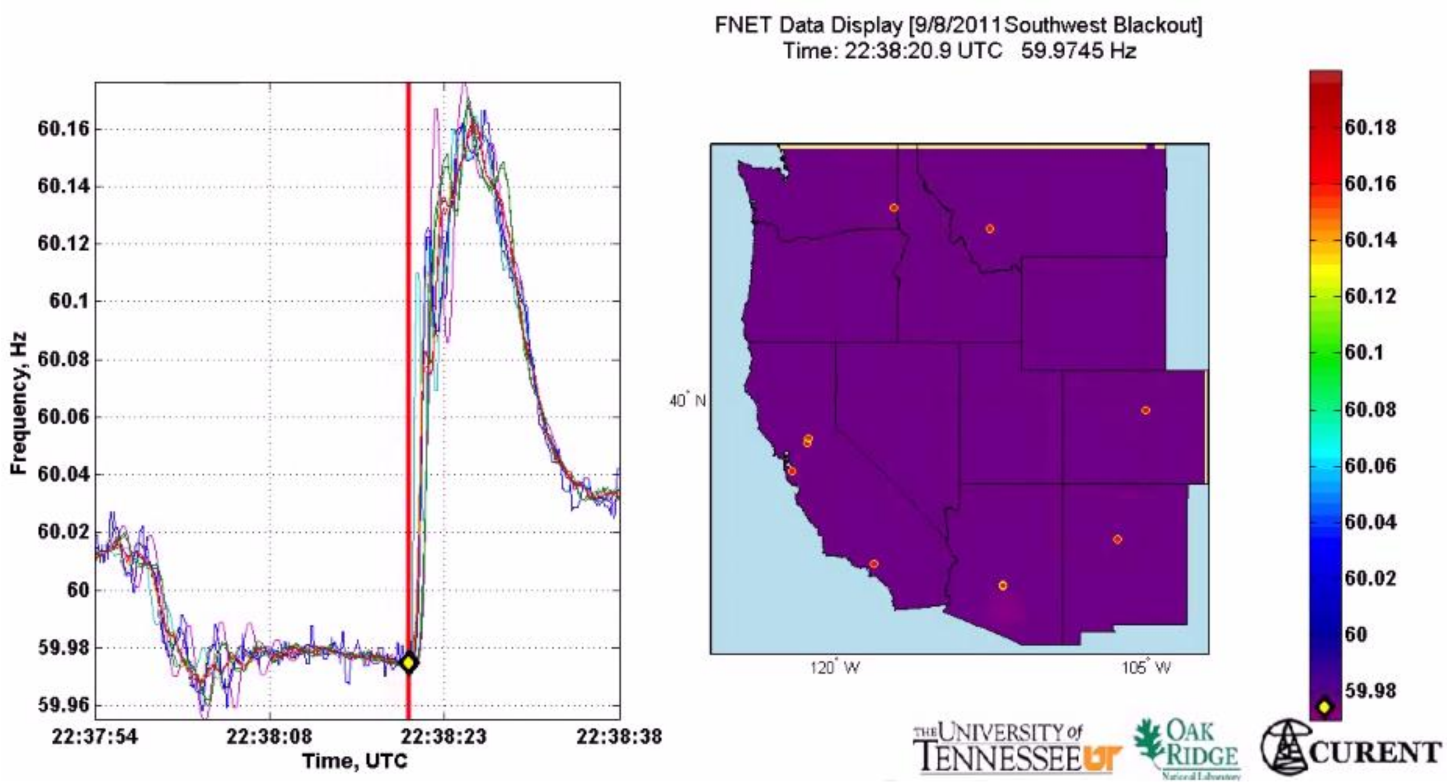

Topcu (Austin)


Zhao (NREL)

Zhao, Topcu, Li, L, TAC 2014
http://netlab.caltech.edu

Motivation

- All buses synchronized to same nominal frequency (US: 60 Hz; Europe/China: 50 Hz)
- Supply-demand imbalance ➔ frequency fluctuation



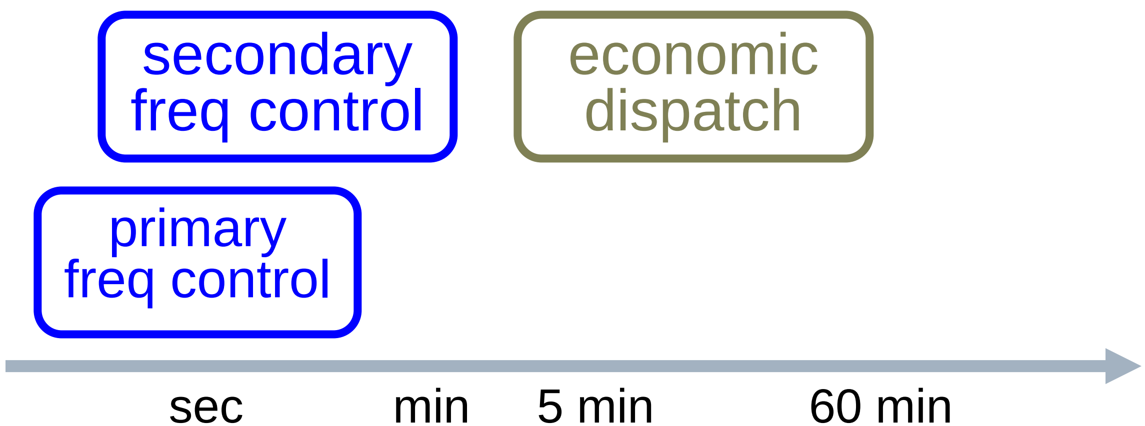
in)

2011 Southwest blackout

Why load-side participation

Ubiquitous continuous load-side control can supplement generator-side control

- faster (no/low inertia)
- more reliable (large #)
- better localize disturbances
- reducing generator-side control capacity



How

How to design load-side frequency control ?

How does it interact with generator-side control ?

Literature: load-side control

Original idea & early analytical work

- Schweppe et al 1980; Bergin, Hill, Qu, Dorsey, Wang, Varaiya …

Small scale trials around the world

- D.Hammerstrom et al 2007, UK Market Transform Programme 2008

Early simulation studies

- Trudnowski et al 2006, Lu and Hammerstrom 2006, Short et al 2007, Donnelly et al 2010, Brooks et al 2010, Callaway and I. A. Hiskens, 2011, Molina-Garcia et al 2011

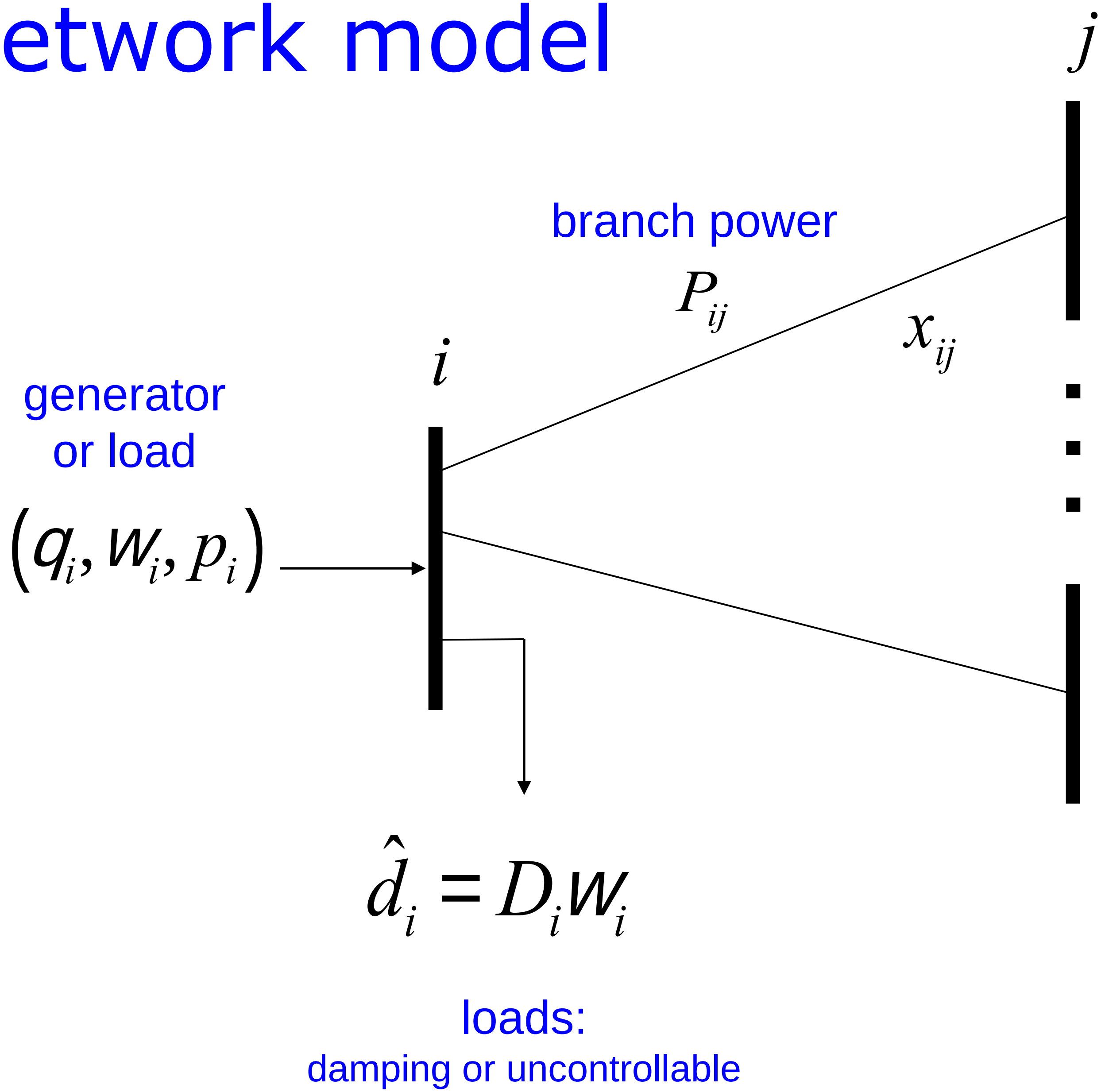
Analytical work – load-side control

- Zhao et al (2012/2014), Mallada and Low (2014), Mallada et al (2014), Zhao and Low (2014), Zhao et al (2015)
- Simpson-Porco et al 2013, You and Chen 2014, Zhang and Papachristodoulou (2014), Ma et al (2014), Zhao, et al (2014),

Recent analysis – generator-side/microgrid control:

- Andreasson et al (2013), Zhang and Papachristodoulou (2013), Li et al (2014, 2016), Burger et al (2014), You and Chen (2014), Simpson-Porco et al (2013), Hill et al (2014), Dorfler et al (2014)

Network model



i : region/control area/balancing authority

Network model

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + p_i - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \to j$$

Generator bus: $M_i > 0$

Load bus: $M_i = 0$

Network model

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + \boxed{p_i} - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \rightarrow j$$

Generator bus: p_i is real power injection

Load bus: p_i is controllable load

Generator-side control

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + \boxed{p_i} - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\, i \to j$$

generator bus:

$$\dot{p}_i = -\frac{1}{\tau_{bi}}\left(p_i + a_i\right)$$

$$\dot{a}_i = -\frac{1}{\tau_{gi}}\left(a_i + p_i^c\right)$$

primary control $p_i^c(t) = p_i^c\left(w_i(t)\right)$

e.g. freq droop $p_i^c\left(w_i\right) = -\,b_i w_i$

Load-side control

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + \boxed{p_i} - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \to j$$

Load bus:

how to design feedback control ?

Network model

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + p_i - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \to j$$

Suppose the system is in steady state

Then: disturbance in gen/load …

Load-side controller design

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + p_i - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \to j$$

Control goals

Zhao, Topcu, Li, Low
TAC 2014
Mallada, Zhao, Low
Allerton, 2014

- Rebalance power & stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows
- Respect line limits

Load-side controller design

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + p_i - \sum_e C_{ie} P_e$$

$$P_{ij} = b_{ij} \sin\left(\theta_i - \theta_j\right) \qquad \forall\ i \rightarrow j$$

Control goals (while min disutility)

Zhao, Topcu, Li, Low
TAC 2014

Mallada, Zhao, Low
Allerton, 2014

- Rebalance power & stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows
- Respect line limits

Load-side controller design

Design control law whose equilibrium solves:

$$
\begin{aligned}
\min_{d,P} \quad & \sum_i c_i(d_i) & & \text{load disutility} \\
\text{s. t.} \quad & P_i^m - d_i = \sum_e C_{ie} P_e \quad \text{node } i & & \text{power balance} \\
& \sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k \quad \text{area } k & & \text{inter-area flows} \\
& \underline{P}_e \le P_e \le \overline{P}_e \quad \text{line } e & & \text{line limits}
\end{aligned}
$$

Control goals (while min disutility)

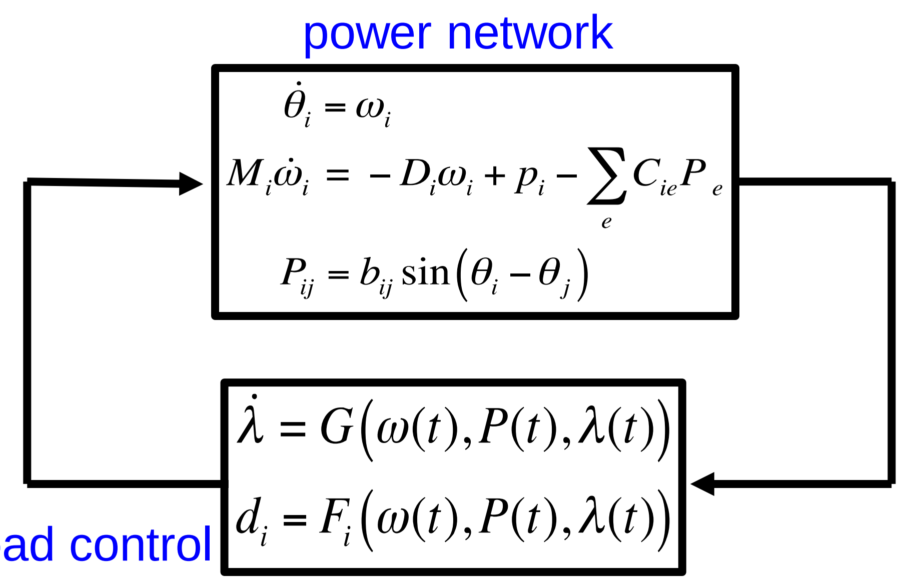
- Rebalance power & stabilize frequency
- Restore nominal frequency
- Restore scheduled inter-area flows
- Respect line limits

freq will emerge as Lagrange multiplier for power imbalance

Load-side controller design

Design control (G, F) s.t. closed-loop system

- is asymptotically stable
- has equilibrium that is optimal

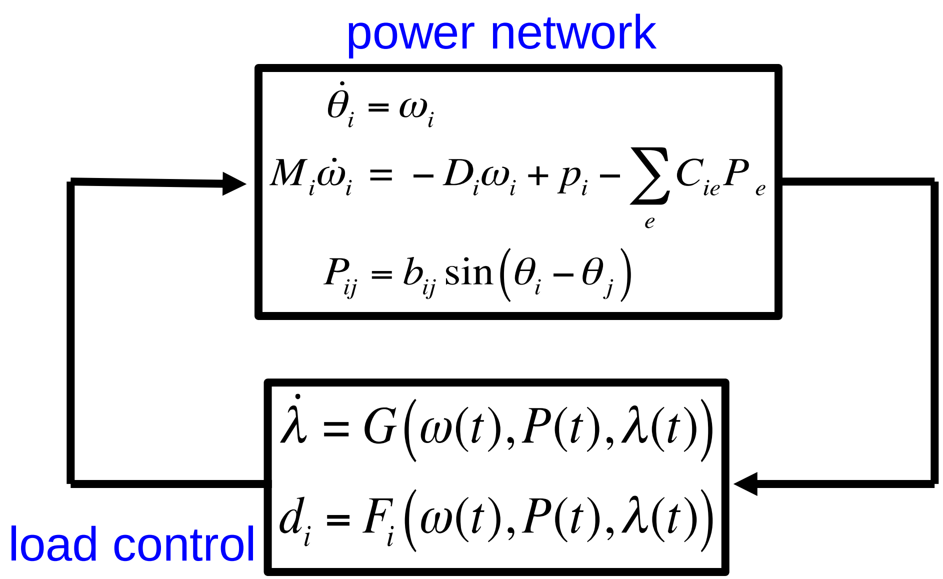


$$
\begin{aligned}
&\min_{d,P} && \sum_i c_i(d_i) && \\
&\text{s. t.} && P_i^m - d_i = \sum_e C_{ie} P_e && \text{node } i \\
& && \sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k && \text{area } k \\
& && \underline{P}_e \le P_e \le \overline{P}_e && \text{line } e
\end{aligned}
$$

Load-side controller design

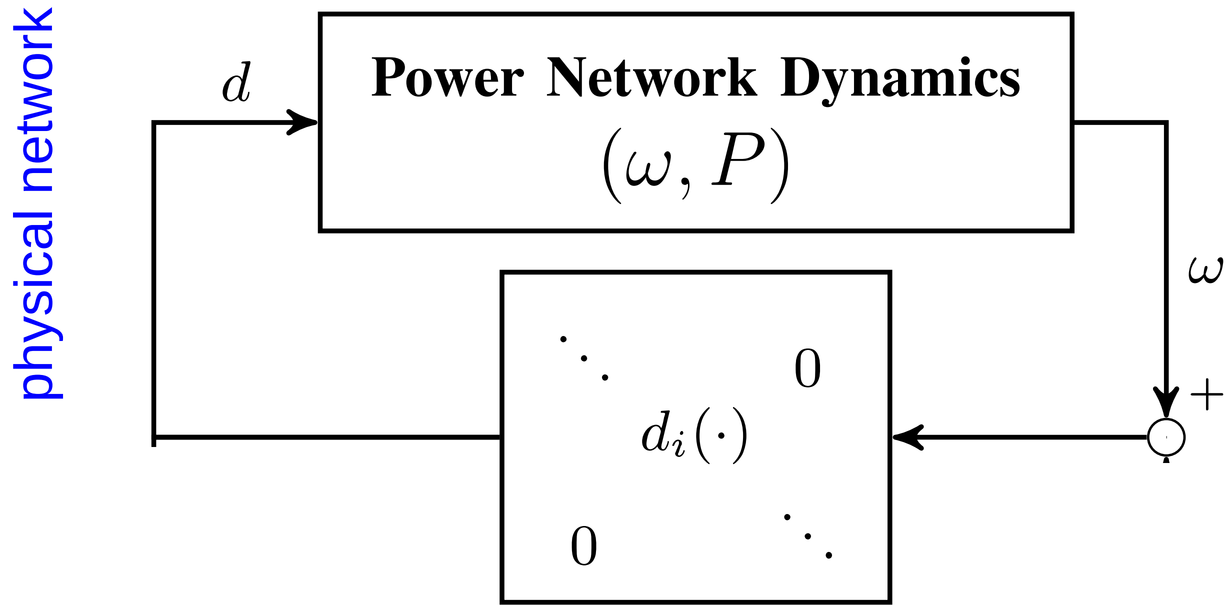
Idea: exploit system dynamic as part of primal-dual algorithm for modified opt

- Distributed algorithm
- Control goals in equilibrium
- Stability analysis



$$
\begin{aligned}
&\min_{d,P} && \sum_i c_i(d_i) && \\
&\text{s. t.} && P_i^m - d_i = \sum_e C_{ie} P_e && \text{node } i \\
& && \sum_{i \in N_k} \sum_e C_{ie} P_e = \hat{P}_k && \text{area } k \\
& && \underline{P}_e \le P_e \le \overline{P}_e && \text{line } e
\end{aligned}
$$

Summary: control architecture

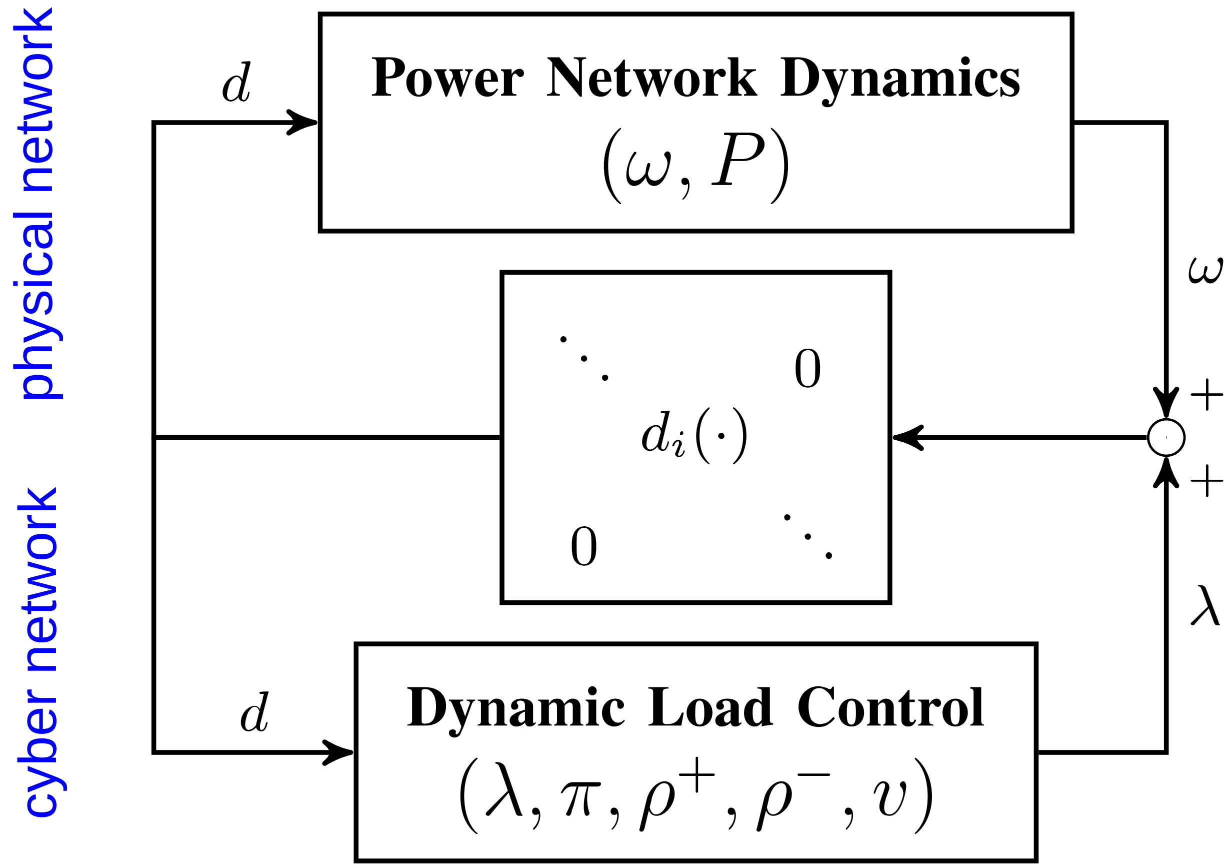


Primary load-side frequency control (linear PF)

- completely decentralized
- Theorem: globally stable dynamic, optimal equilibrium

Zhao, Topcu, Li, Low. TAC 2014

Summary: control architecture

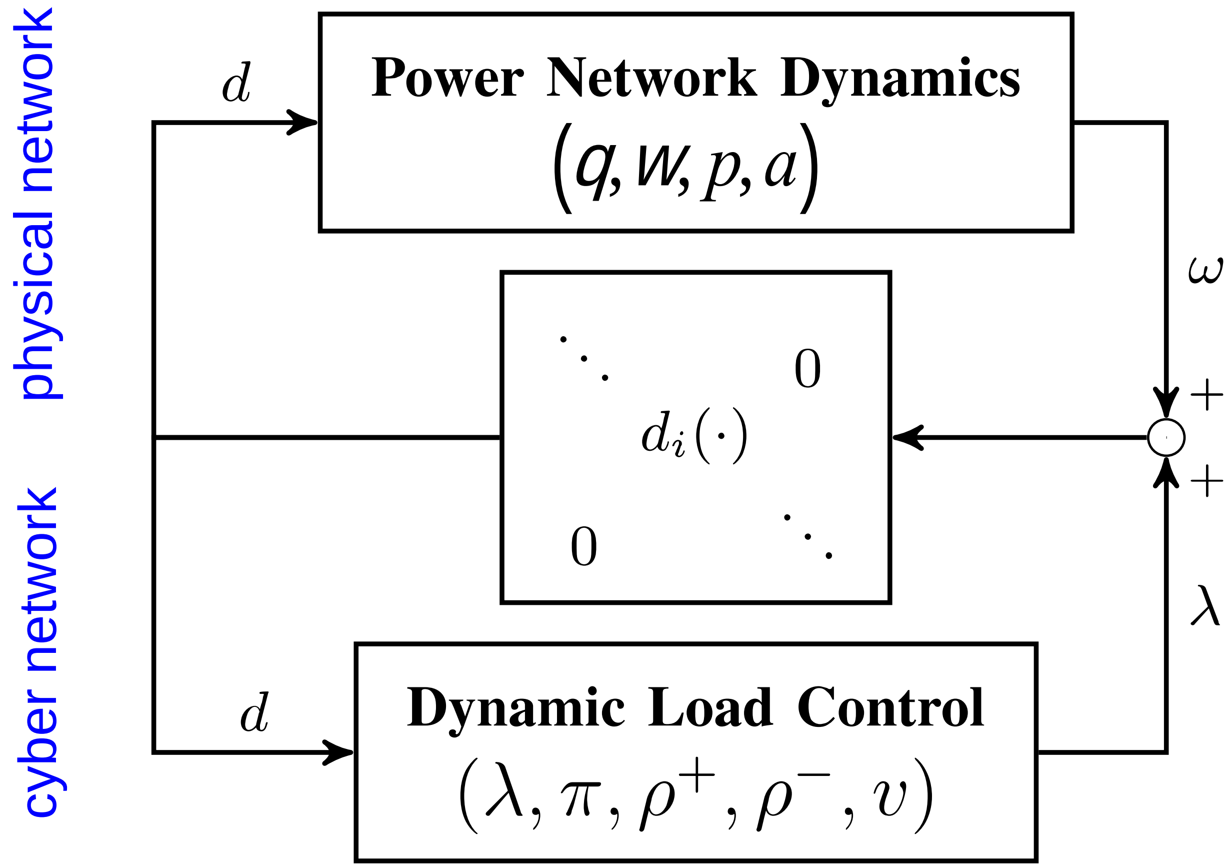


Secondary load-side frequency control (linear PF)

- communication with neighbors
- <u>Theorem</u>: globally stable dynamic, optimal equilibrium

Mallada, Zhao, Low. Allerton 2014

Summary: control architecture



With **generator-side** control, **nonlinear** power flow

- load-side improves both transient & eq
- Theorem: stable dynamic, optimal equilibrium

Zhao, Mallada, Low. CISS 2015

Simulations

Dynamic simulation of IEEE 39-bus system

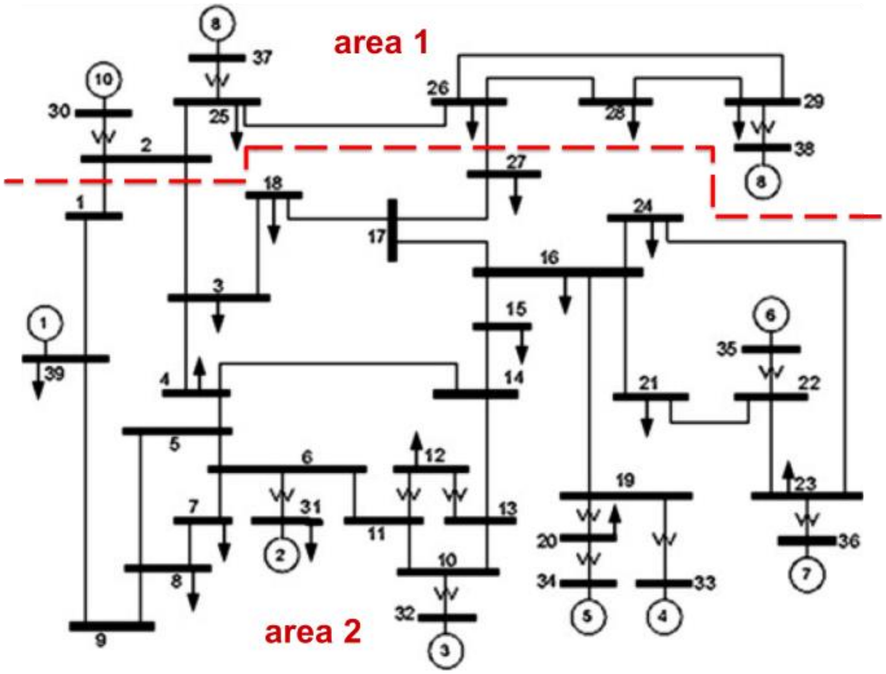
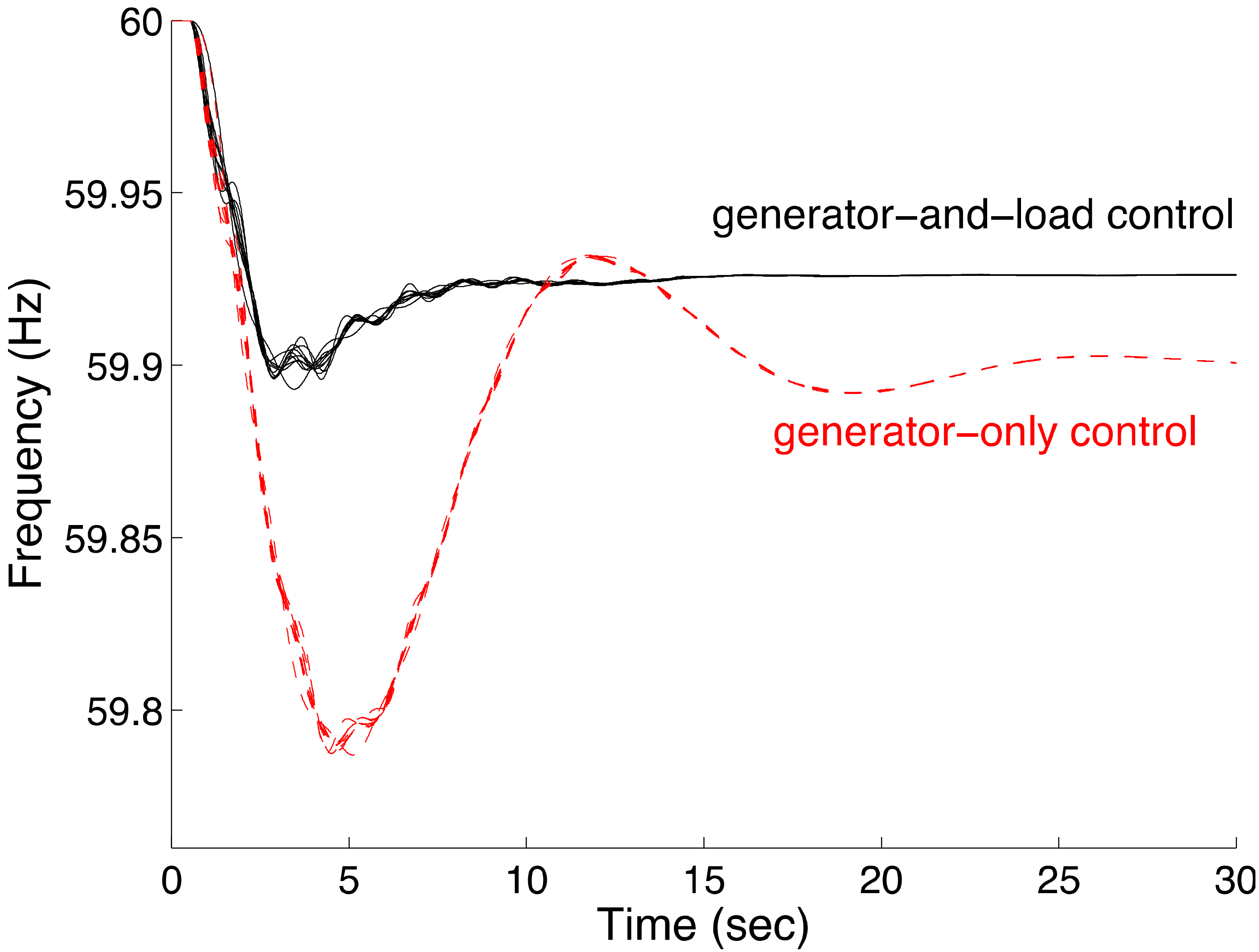


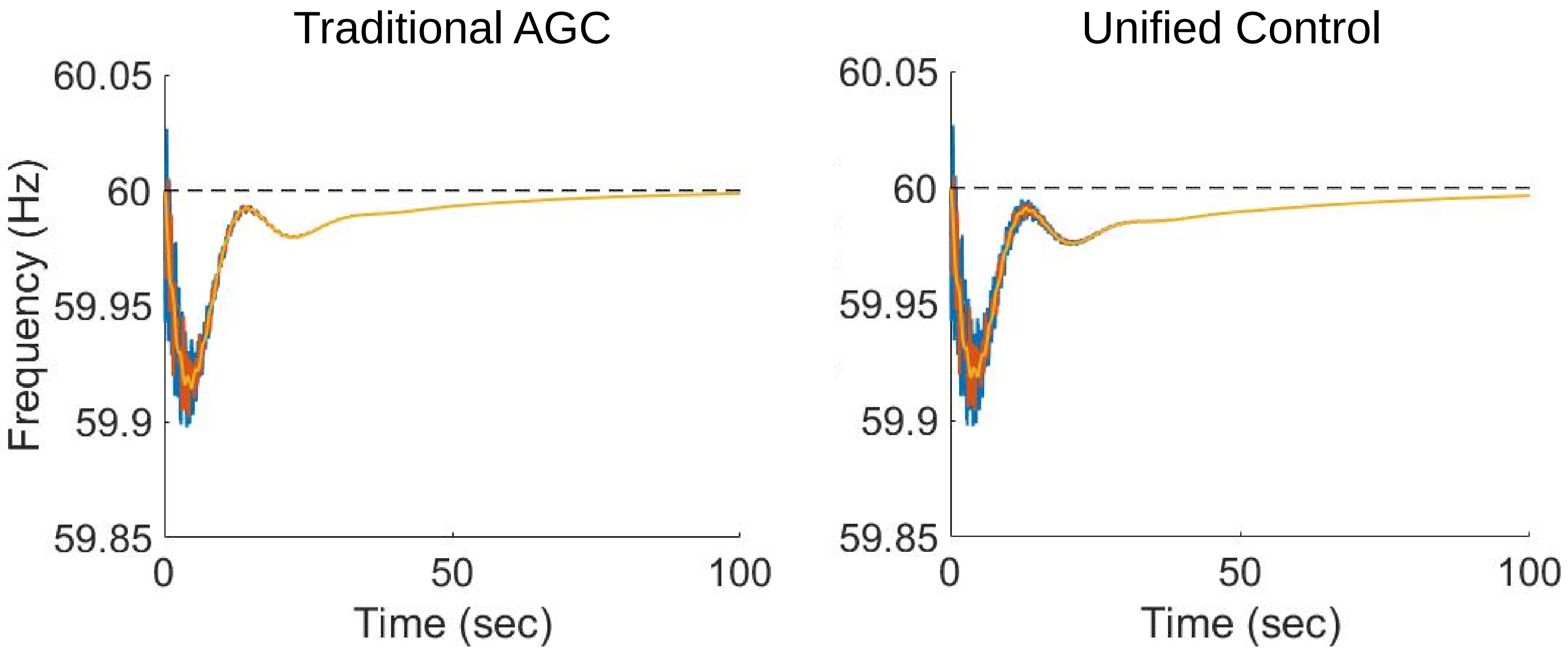
Fig. 2: IEEE 39 bus system : New England

- Power System Toolbox (RPI)
- Detailed generation model
- Exciter model, power system stabilizer model
- Nonzero resistance lines

Primary control

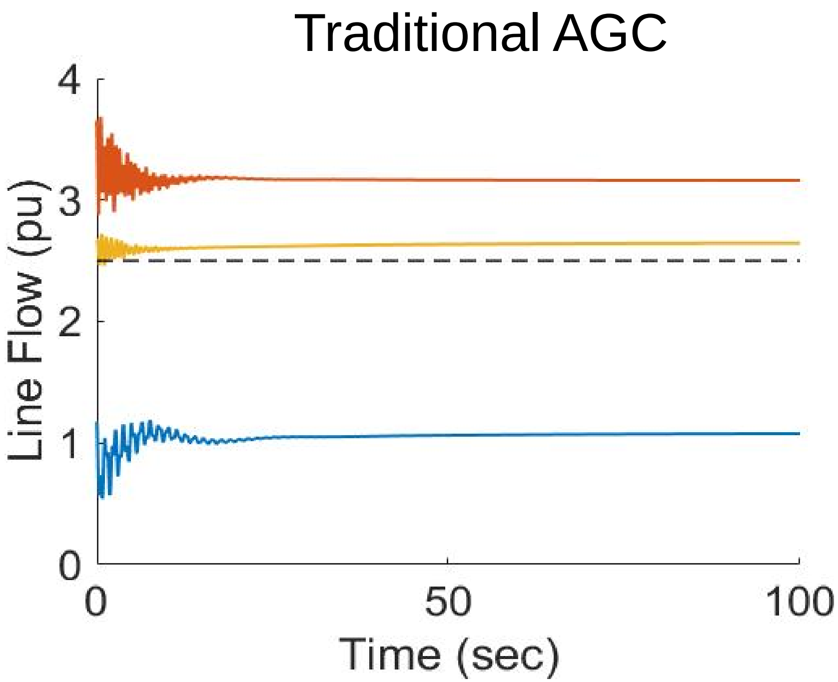


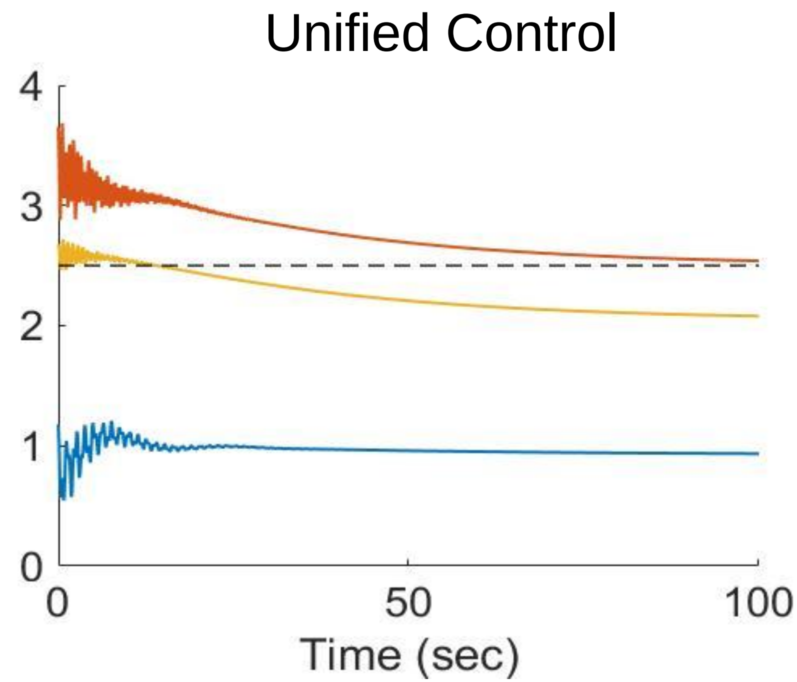
Secondary control



Similar shape but local frequencies differ more, higher control effort, slightly longer settling time

Secondary control





AGC blind to line limits

Unified Control enforces line limits

Key message

Large network of DERs

- Real-time optimization at scale
- Computational challenge: power flow solution

Online optimization [feedback control]

- Network computes power flow solutions in real time at scale for free
- Exploit it for our optimization/control
- Naturally adapts to evolving network conditions

Examples

- Slow timescale: OPF
- Fast timescale: frequency control