Adaptive Charging Network

Steven Low

CMS, EE

Caltech

January 2023

Acknowledgement

G. Lee

C. Jin

T. Lee

D. Lee

Acknowledgement

powerflex
EDF renewables

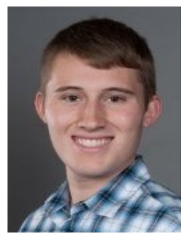
Caltech

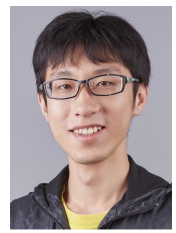

G. Lee


C. Jin

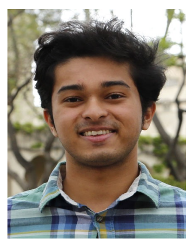

T. Lee


D. Lee

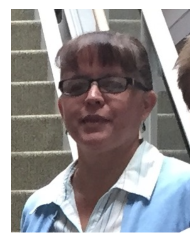

Z. Lee


T. Li


J. Pang


S. Sharma


D. Guo


C. Ortega


R. Lee


D. Chang


Z. Low, Cornell


K. Erliksson, Lund


D. Johansson, Lund

and many others …

Agenda

ACN: Caltech testbed

- Testbed to commercial deployment

ACN: Research Portal

- Data, Sim, Live

ACN: pricing demand charge

- Monthly billing at workplaces

Unbalanced 3-phase modeling

- Motivation, 3-phase network models

ACN testbed

IEEE TRANSACTIONS ON SMART GRID, VOL. 12, NO. 5, SEPTEMBER 2021 4339

Adaptive Charging Networks: A Framework for Smart Electric Vehicle Charging

Zachary J. Lee, *Graduate Student Member, IEEE*, George Lee, Ted Lee, Cheng Jin, Rand Lee, Zhi Low, Daniel Chang, Christine Ortega, and Steven H. Low, *Fellow, IEEE*

2016 GlobalSIP Conference:

Adaptive Charging Network for Electric Vehicles

George Lee[1, 2], Ted Lee[2], Zhi Low[3], Steven H. Low[2], and Christine Ortega[2]

[1]PowerFlex Systems
[2]Division of Engineering & Applied Science, Caltech
[3]Math Department, Cornell

ACN Research Portal

2019 ACM e-Energy:

ACN-Data: Analysis and Applications of an Open EV Charging Dataset

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IEEE TRANSACTIONS ON SMART GRID, VOL. 12, NO. 6, NOVEMBER 2021 5113

ACN-Sim: An Open-Source Simulator for Data-Driven Electric Vehicle Charging Research

Zachary J. Lee, Sunash Sharma, Daniel Johansson, and Steven H. Low, *Fellow, IEEE*

ACN Pricing

ELSEVIER

Electric Power Systems Research

Volume 189, December 2020, 106694

Pricing EV charging service with demand charge ☆

Zachary J. Lee [a], John Z.F. Pang [b], Steven H. Low [a, b]

PSCC 2020

Unbalance 3-phase modeling

Power System Analysis

A Mathematical Approach

Steven H. Low

DRAFT available at: http://netlab.caltech.edu/book/

Corrections, questions, comments appreciated!

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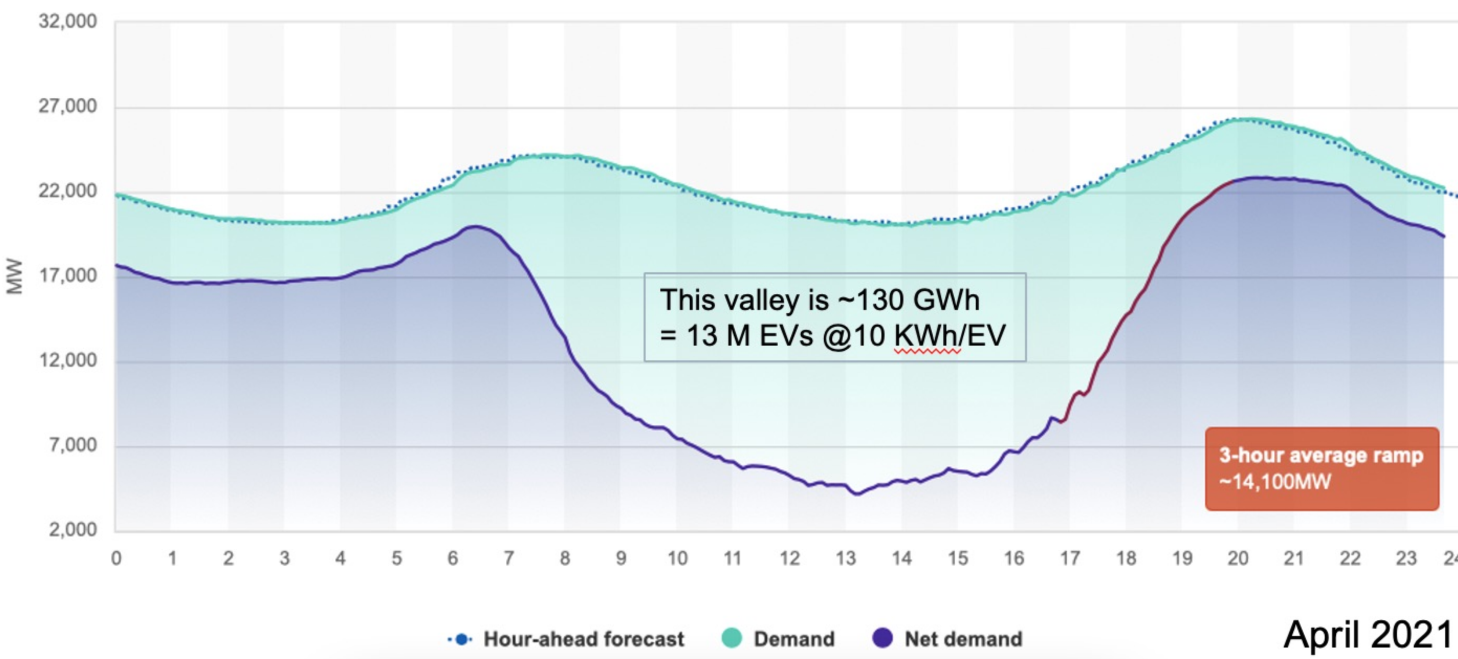
Workplace charging

CA commitment

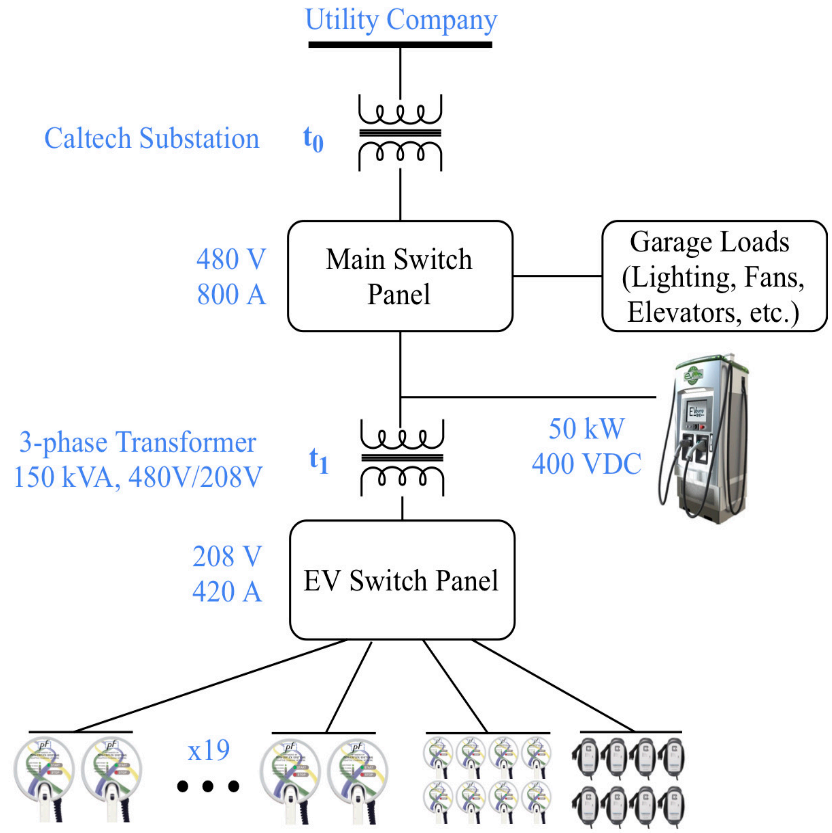
- ~~50%~~ 60% renewables by 2030, 100% by 2045
- 1.5M ZEV by 2025, 5M by 2030 (CA has ~15M cars)



Drivers <u>twice as likely</u> to get EV when workplace charging is available

(EDF Renewables survey Feb 2018)

Caltech ACN: physical system



Caltech ACN: cyber system

Model predictive control: QCQP

$$
\begin{aligned}
\min_{r \geq 0} \quad & C(r) \\
\text{subject to} \quad & r_i(t) \leq \overline{r}_i(t) \qquad \forall i,\ \forall t \\
& \sum_t r_i(t)\,\delta \;=\; e_i \quad \forall i \\
& \sum_i r_i(t) \;\leq\; P(t) \quad \forall t
\end{aligned}
$$

Highly customizable QCQP

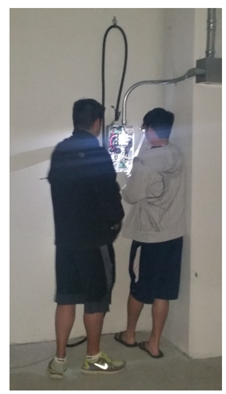
- objectives: cost, PV, asap, regularizatn
- constraints: energy, deadlines, capacities
- determine charging rates for all EVs

First deployment Feb 19, 2016

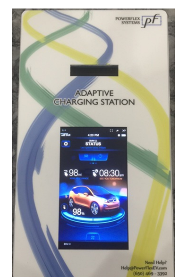
Online optimization of electric vehicle charging

- Enables mass deployment at lower capital & operating costs
- First pilot @Caltech: 54 adaptive programmable chargers
- 2x 150kVA transformers, breakers, grid sensors, etc

debugging

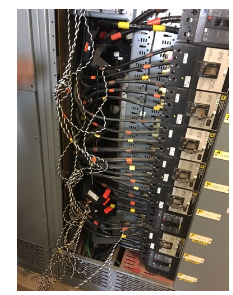


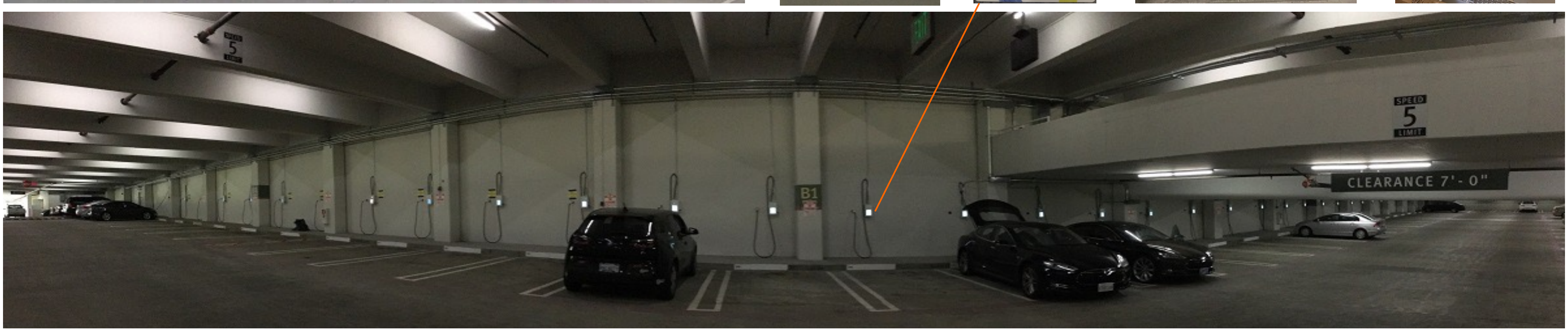
charger



transformer
& subpanels

main
panel





2020

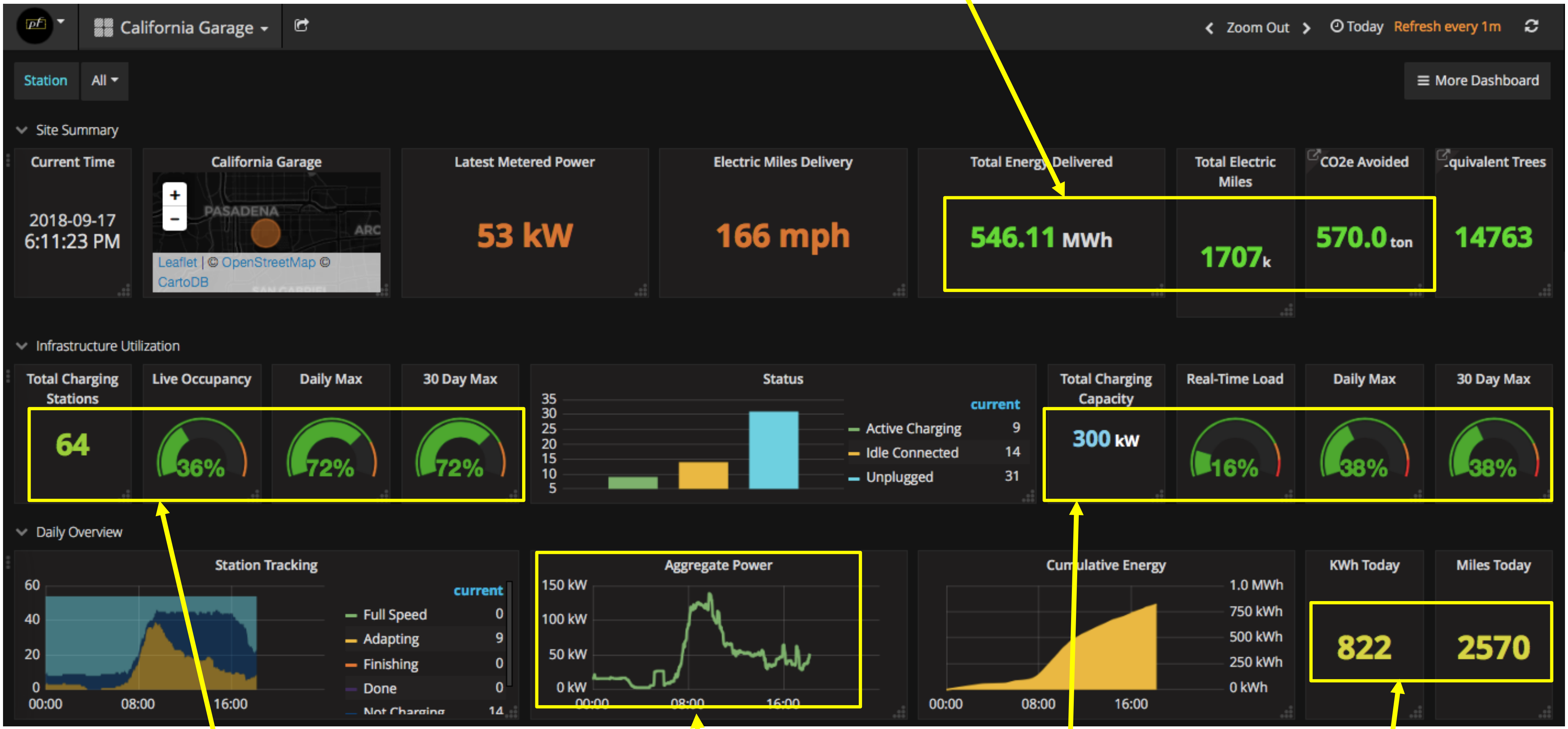
Figure 1. Photos of the N_Wilson_Garage_01 ACN, which is one of the charging sites used to collect data.

The ACN Research Portal has three parts:
(1) ACN-Data: a dataset of over 80,000 EV charging sessions (March 2021)
(2) ACN-Sim: an open-source, data-driven simulation environment
(3) ACN-Live: a framework for field testing algorithms on physical hardware

March 2021: ACN includes a total of 207 level-2 EVSEs and six DC Fast Chargers (DCFC), and covers seven sites at Caltech, NASA's Jet Propulsion Laboratory, a LIGO research facility, and an office building in Northern California.

Caltech ACN

energy delivered & impact to date



charging station utilization

peak power

power utilization

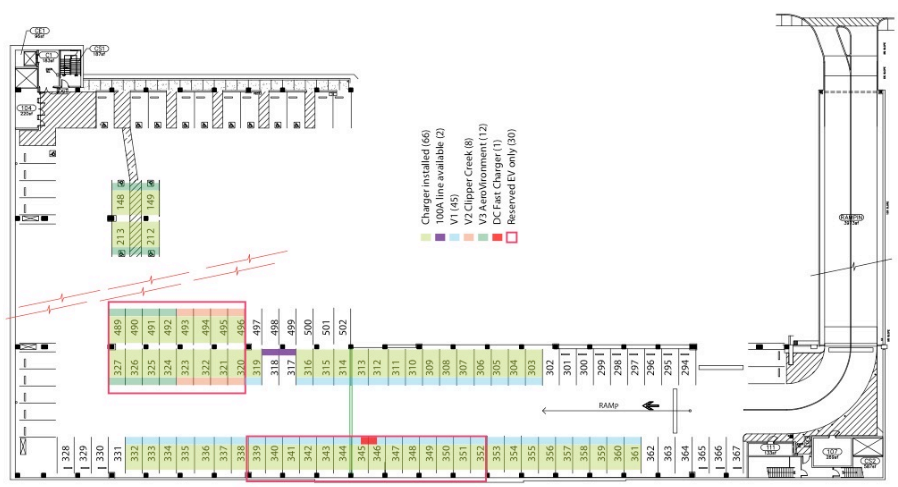
today's energy delivered

Caltech ACN snapshot Sept 17, 2018

Caltech ACN

Spatial utilization snapshot (June 1 – August 31, 2018)

	total	per day	per space	remark
#parking spaces	53			
#days (June 1 – Aug 31, 2018)	92			inc. weekends
#charging sessions	6,103	66	115	>1 session /space/day
occupancy (space-day)	3,374	37	64	69% occupancy
energy delivered (kWh)	54,562	593	1,029	11 kWh /space/day
#hours occupied	28,407	309	536	5.8 hours /space/day



Caltech ACN

- CA Garage operational since 2016
- Delivered 1 GWh (by July 2020, CA)
- Equivalent to 3.2M miles, 1,000 tons of avoided CO2e

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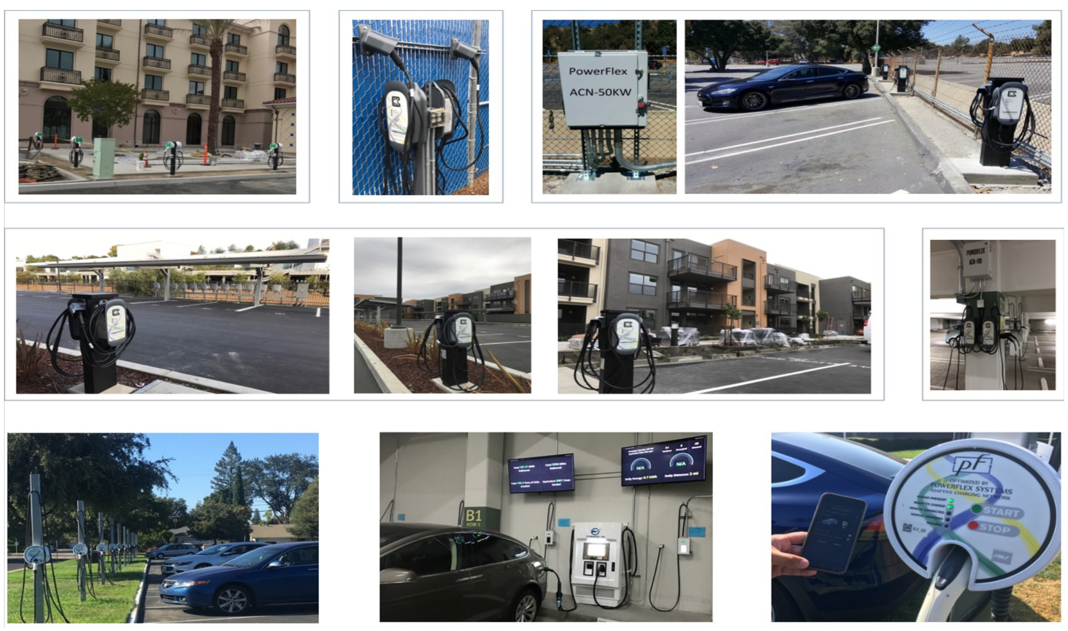


powerflex
EDF renewables

Feb 2020

2,000+
EV CHARGING STATIONS DEPLOYED

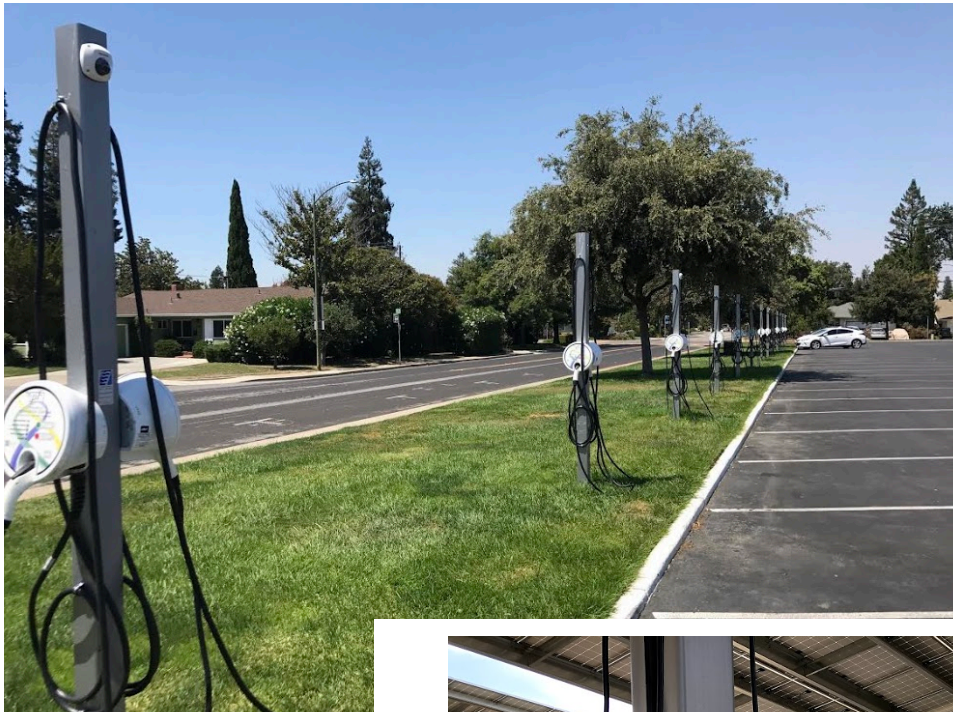
10,000,000+
ELECTRIC MILES DELIVERED SAFELY

(US wide)

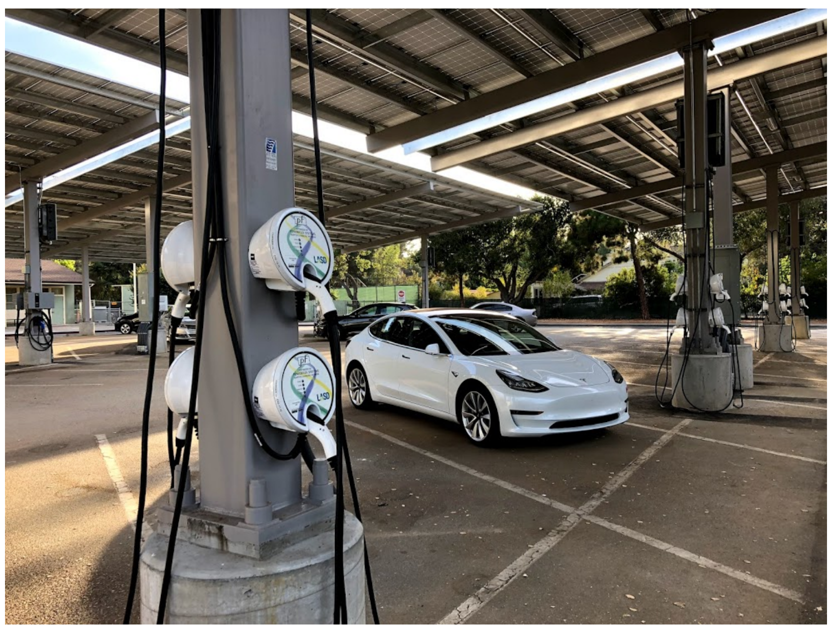
NREL, Golden CO

120 EVSEs

Bay Area high schools

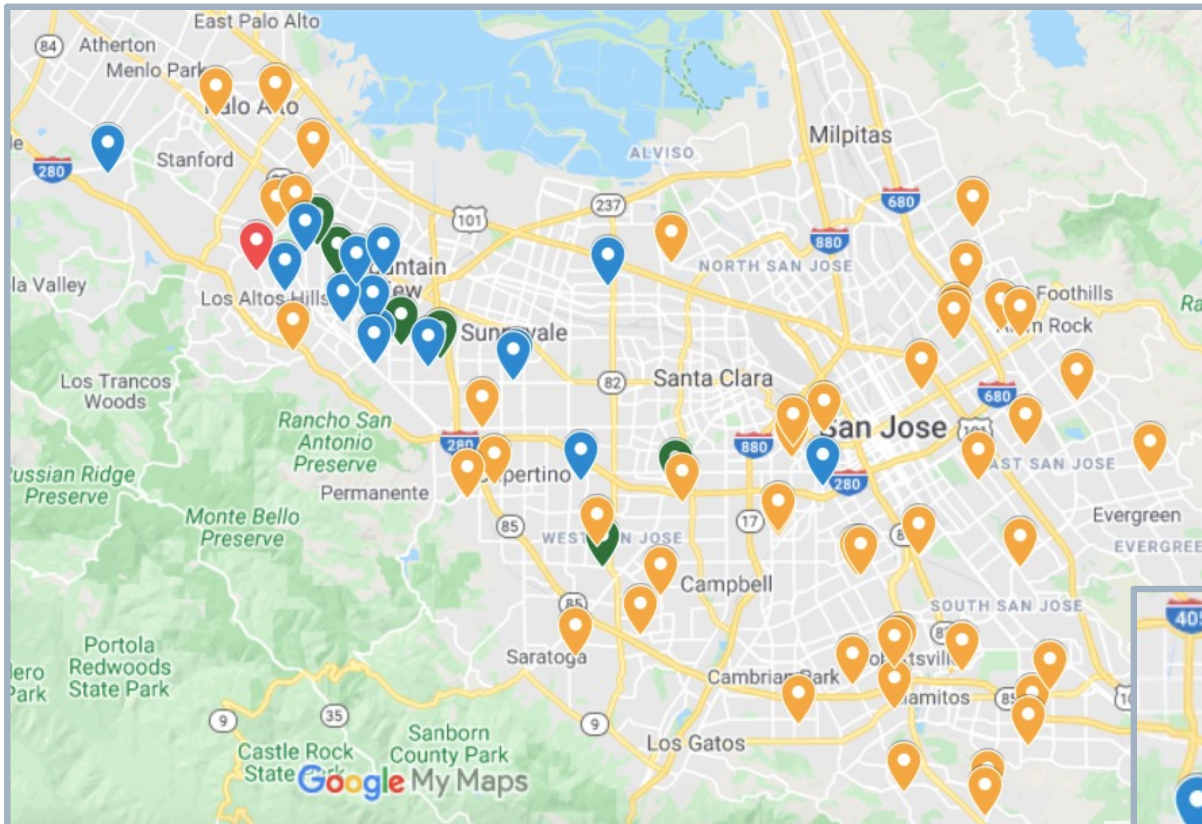


DCFC

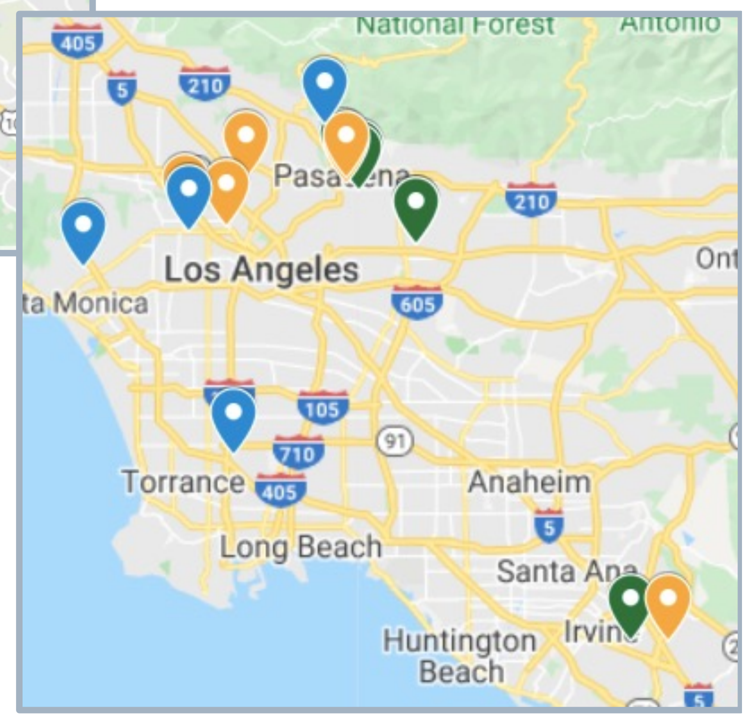


Onsite PV

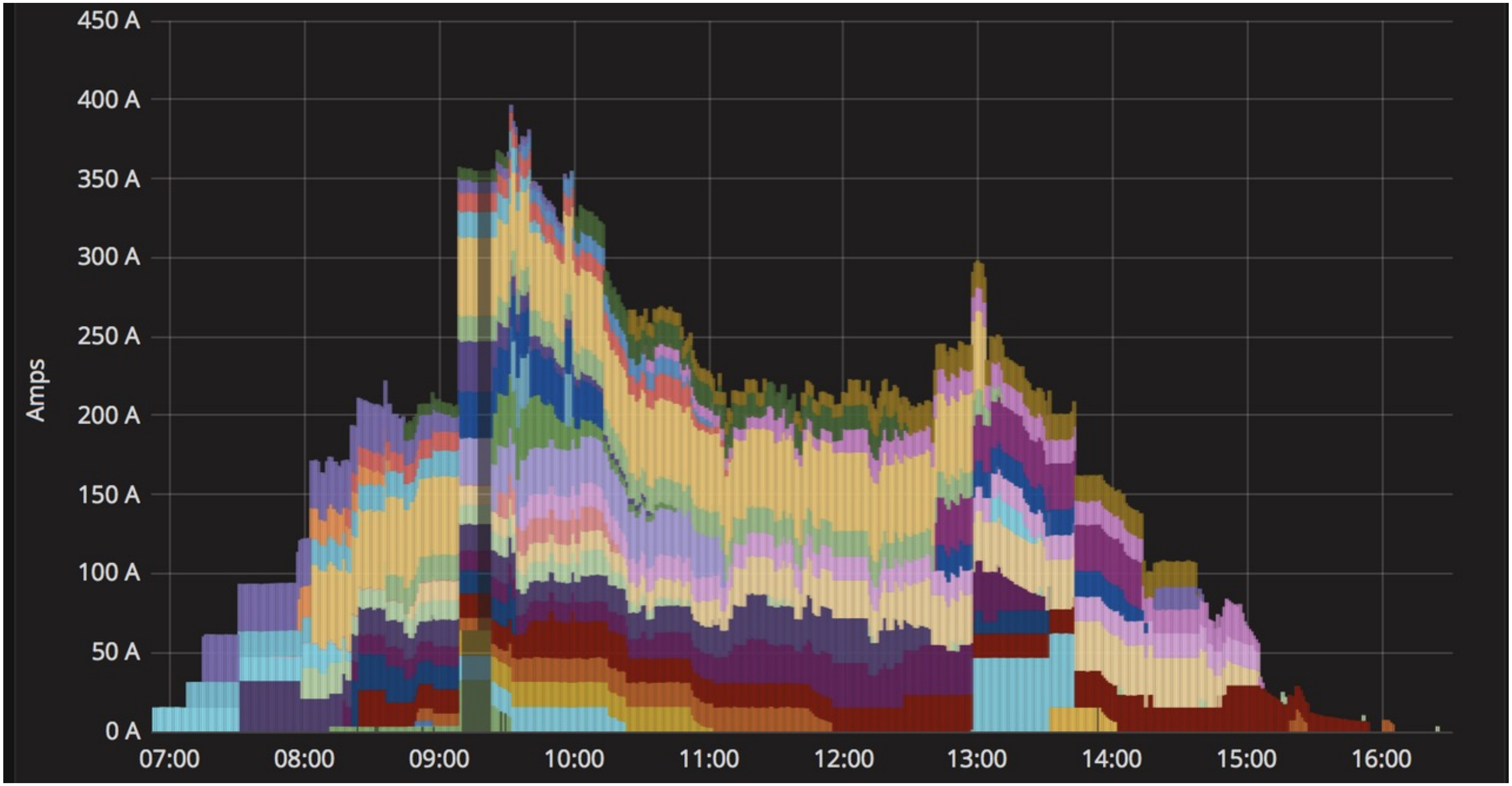
Deployment in CA



powerflex EDF renewables deployment, Sept 2018

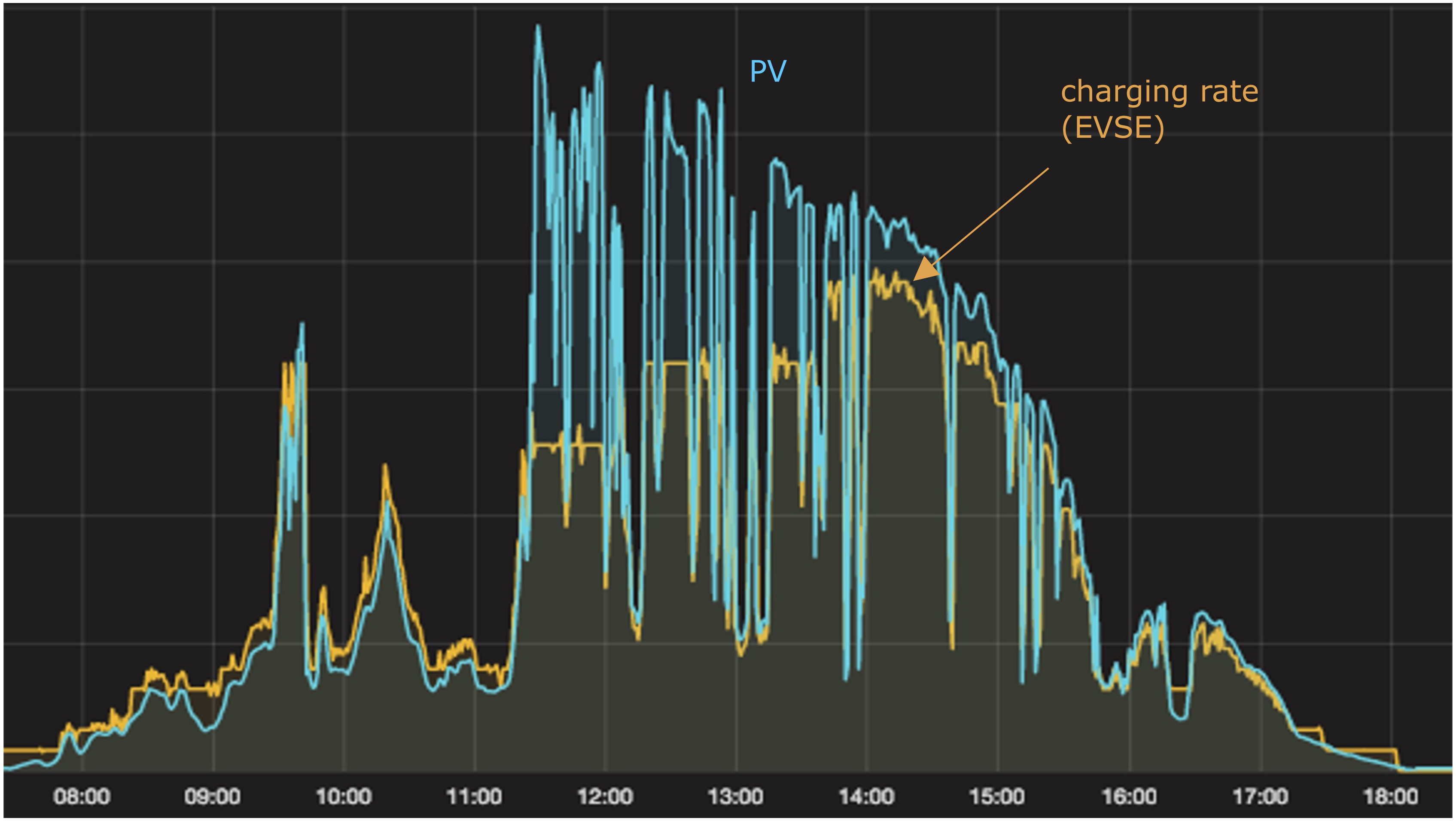


Adaptive charging



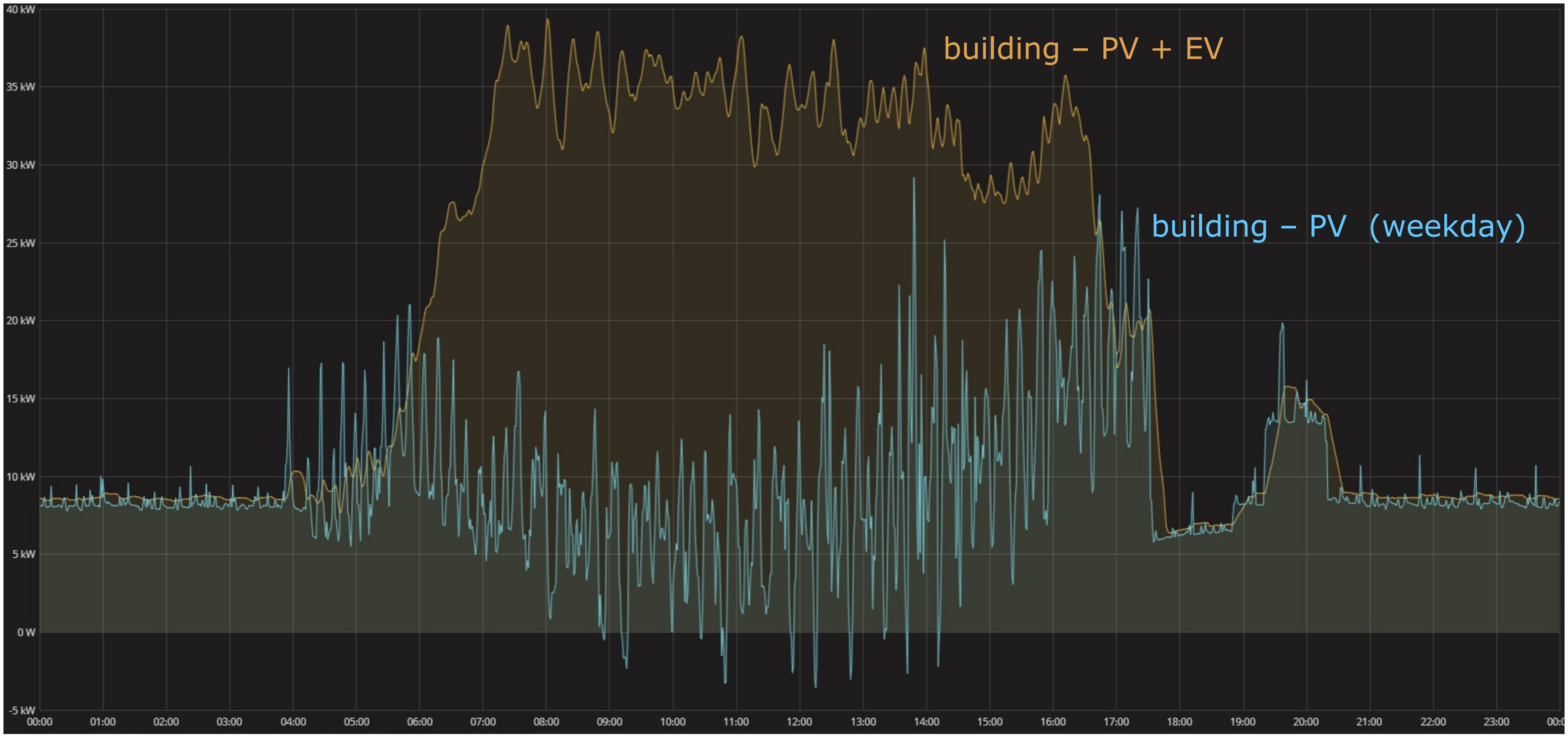
Caltech Jan 2018

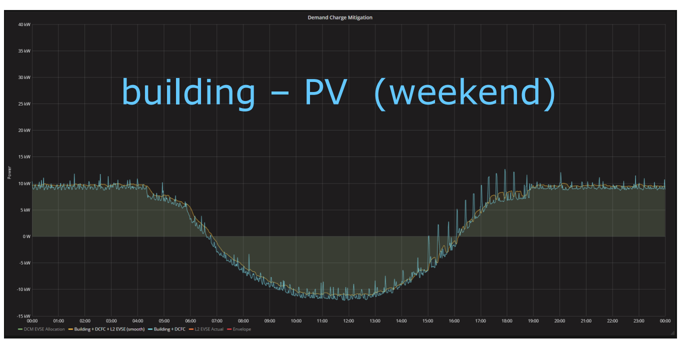
Online tracking



Real-time tracking of PV generation at JPL (10/2016)

Duck Curve & DCM



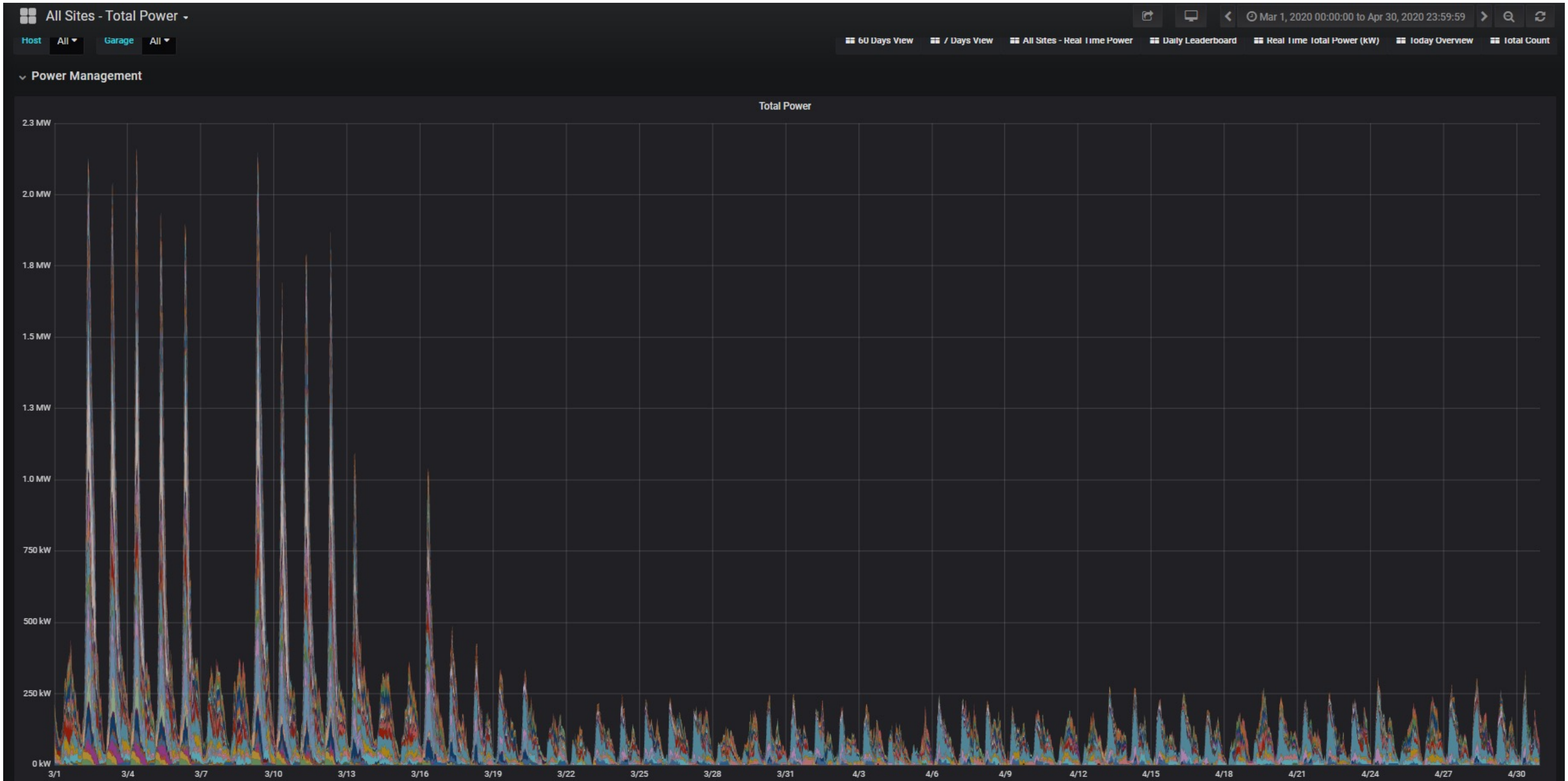


Weekend Duck Curve: building load (10kW) – PV

NREL: demand charge mitigation (Nov 2018)

- Fill Duck Curve valley and maintain net load between 30 kW – 40 kW
- On weekdays: building load is much higher and much more volatile

COVID hit



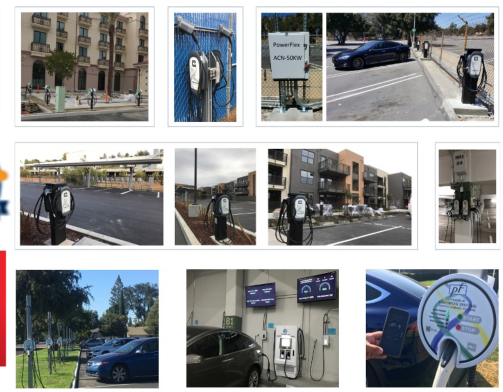
weekdays

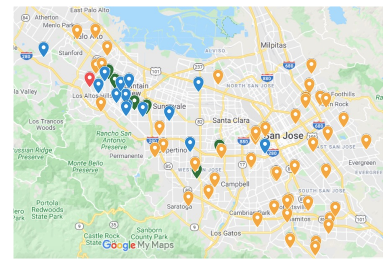
weekdays

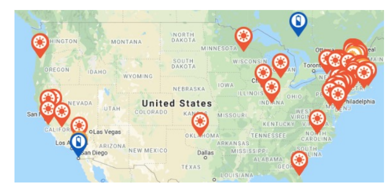
March 16 Monday, 2020

Commercialization: timeline

2010 | 2016 | 2017 | 2019 | PF: EV + solar + storage 2021







Energy mgt **research**

Incubation to **tech transfer**

Scalable **business**

Business case: lower capital cost

Table ES.1: Projections for Statewide PEV Charger Demand

Demand for L2 Destination (Workplace and Public) Chargers (The Default Scenario)			
	Total PEVs	Lower Estimate (Chargers)	Higher Estimate (Chargers)
As of 2017	239,328	21,502	28,701
By 2020	645,093	53,173	70,368
By 2025	1,321,371	99,333	133,270

100,000 Chargers @$15k/ea = $1.5B

$15k/charger is unsustainable

CA CEC & IOU incentive program estimated ~$15k/charger (inc. make ready)

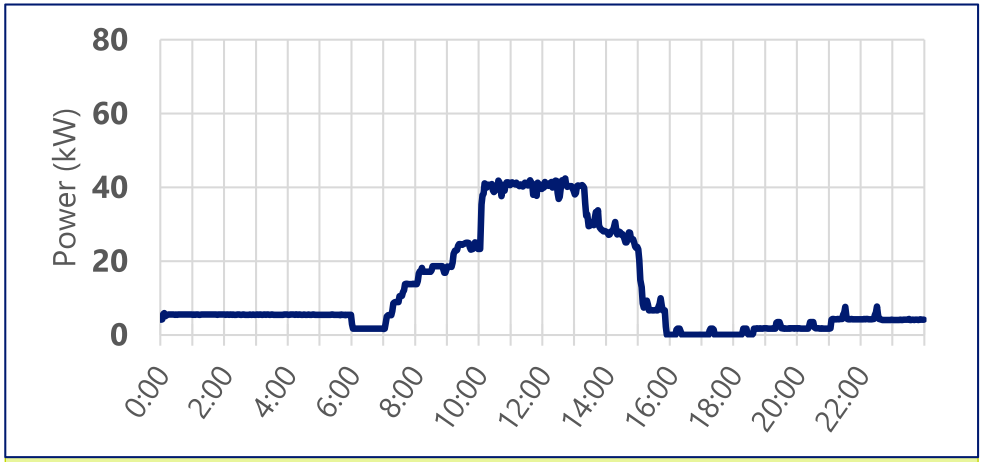
CEC 3/2018 Staff Report

- 168 chargers
  - 118x Universal (J1772) x 6.6kW
  - 50x Tesla x 16kW
- 1.578MW nameplate
  - Connected to 800A/480V panel (max load @80% = 522kW)
  - 3x capacity
  - No Interconnection Upgrade
- Cost: <$3,000/station

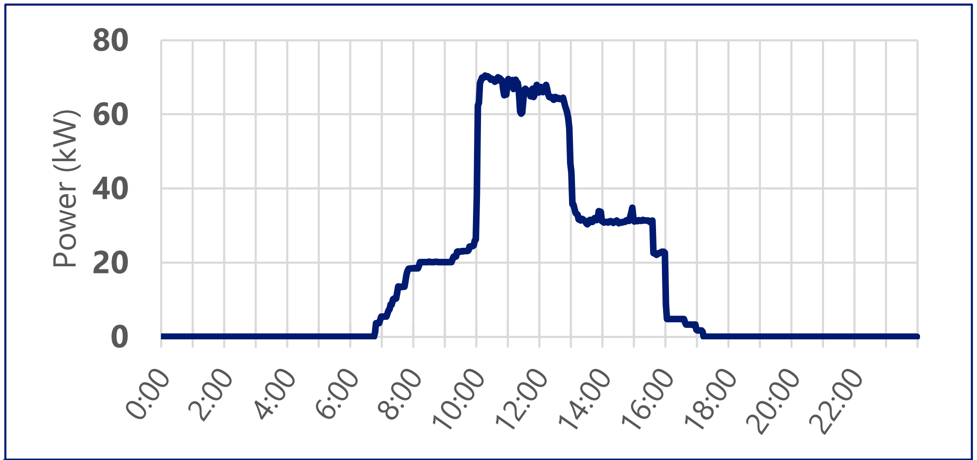
PowerFlex case study: <$3k/charger (inc. make ready)

powerflex EDF renewables 2/2020

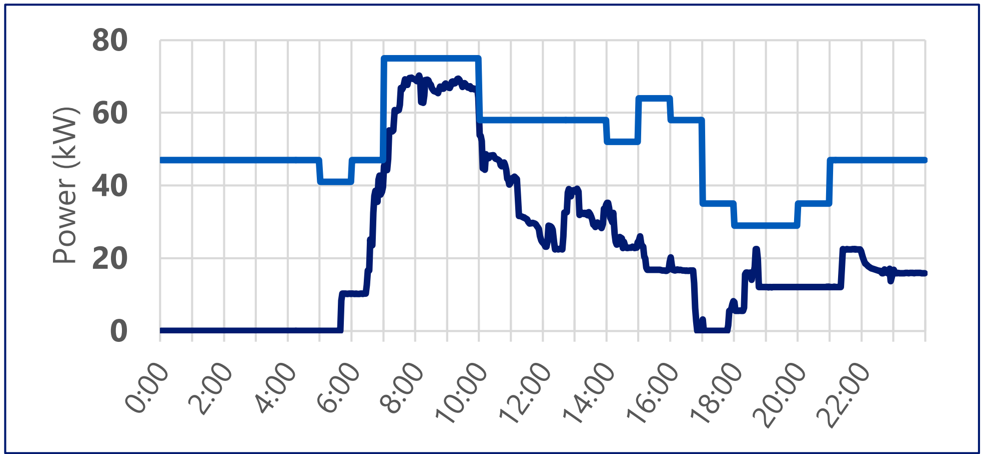
Business case: lower operating cost



Peak Reduction: Reduced Peak by 40% (72kW to 42kW) while still delivering same amount of energy



10am Floodgates: Charging maximized to transformer limits during 10am-2pm to optimize for incentives for consuming surplus solar energy



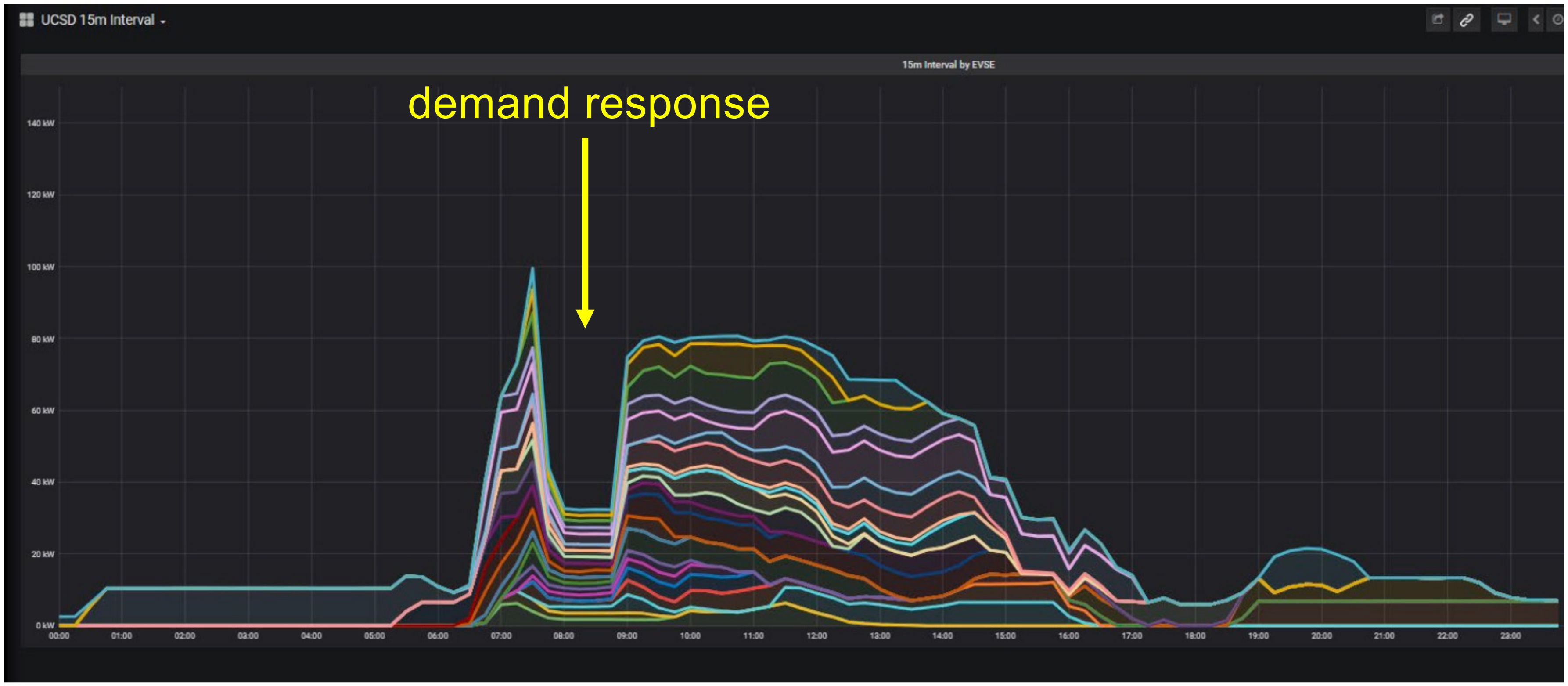
LCFS Curve Following: Charging optimized under LCFS Time-of-Use Value curve

3 ways to reduce operating cost

- Demand charge reduction
- Price arbitrage on ToU tariff
- Increasing LCFS revenue
- EDF – Athena (San Diego, CA)

powerflex EDF renewables 2/2020

Business case: grid services



powerflex EDF renewables 2/2020

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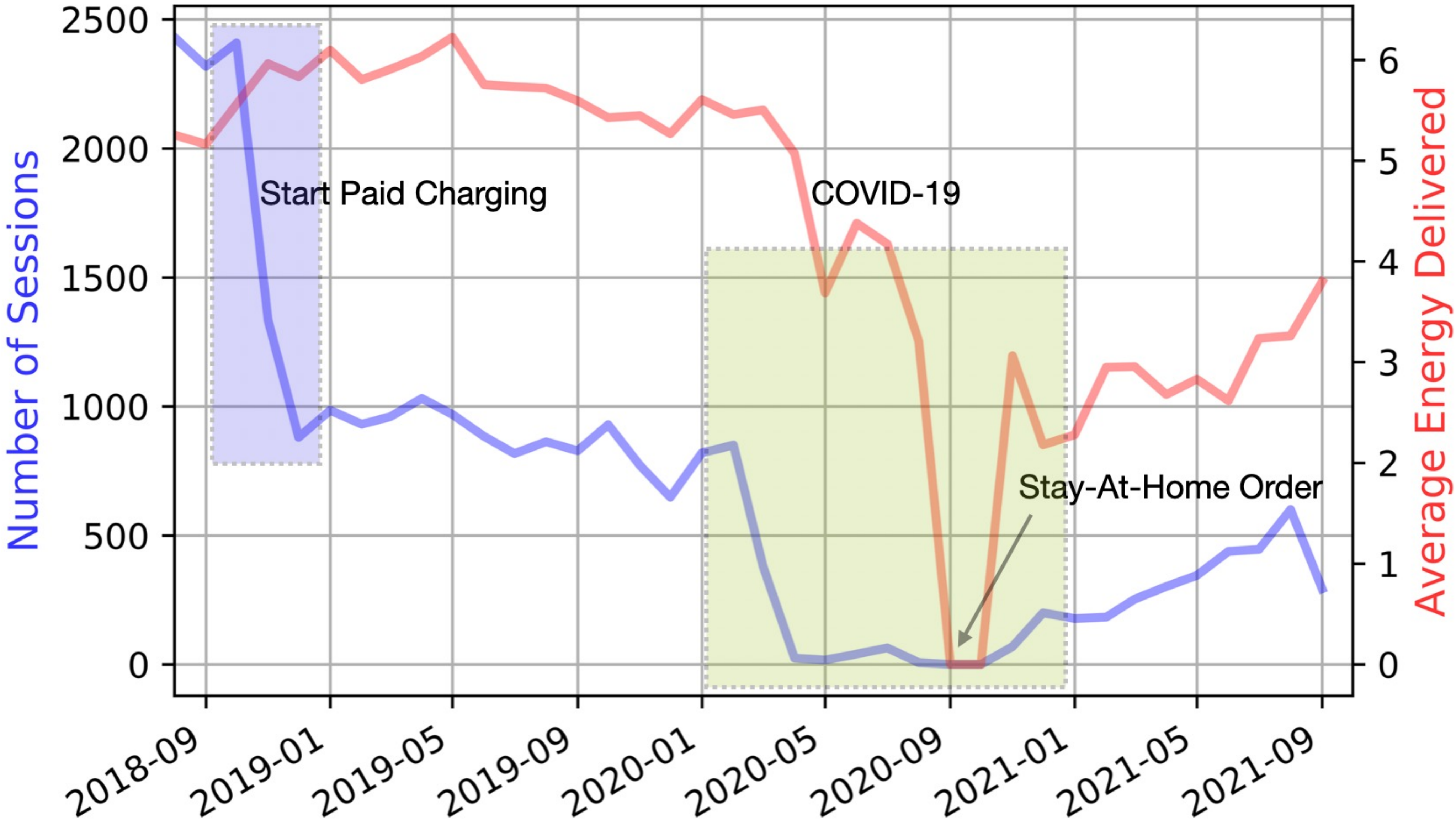
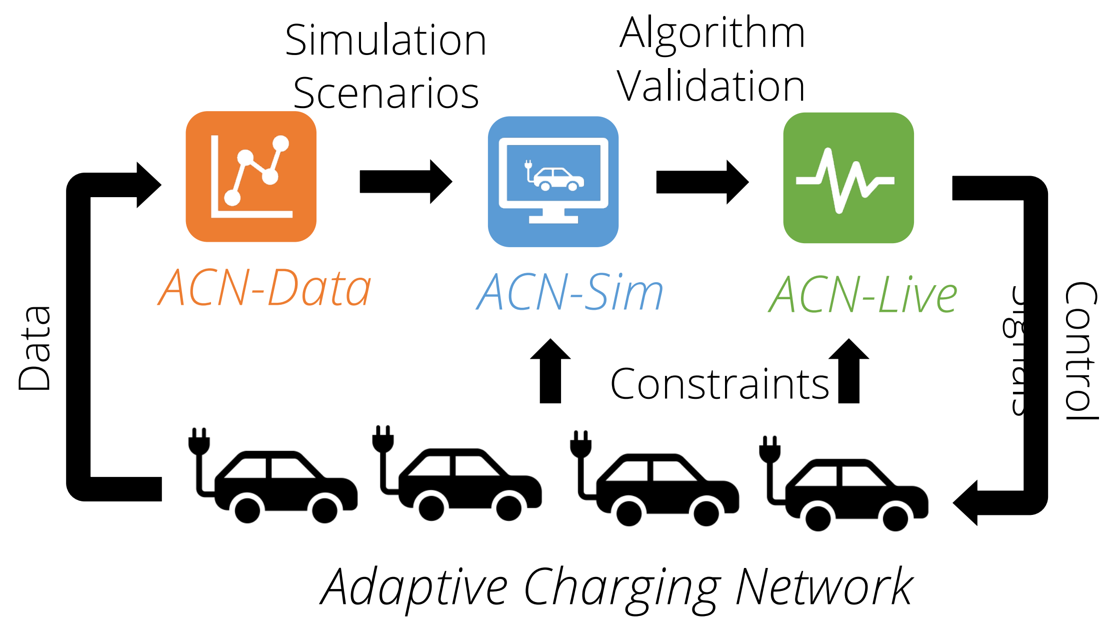
Caltech ACN



Number of Sessions
Average Energy Delivered
Start Paid Charging
COVID-19
Stay-At-Home Order
2500
2000
1500
1000
500
0
6
5
4
3
2
1
0
2018-09
2019-01
2019-05
2019-09
2020-01
2020-05
2020-09
2021-01
2021-05
2021-09

ACN research portal

- ACN-Data
- ACN-Sim
- ACN-Live (HW-in-the-loop)



Lee, Li, Low. ACN-Data: analysis and applications of an open EV charging Dataset
ACM e-Energy, June 2019

Lee, Johansson, Low. ACN-Sim: an open-source simulator for data-driven EV charging research
IEEE SmartGridComm, October 2019

ACN-Data

Caltech, JPL, Bay Area office

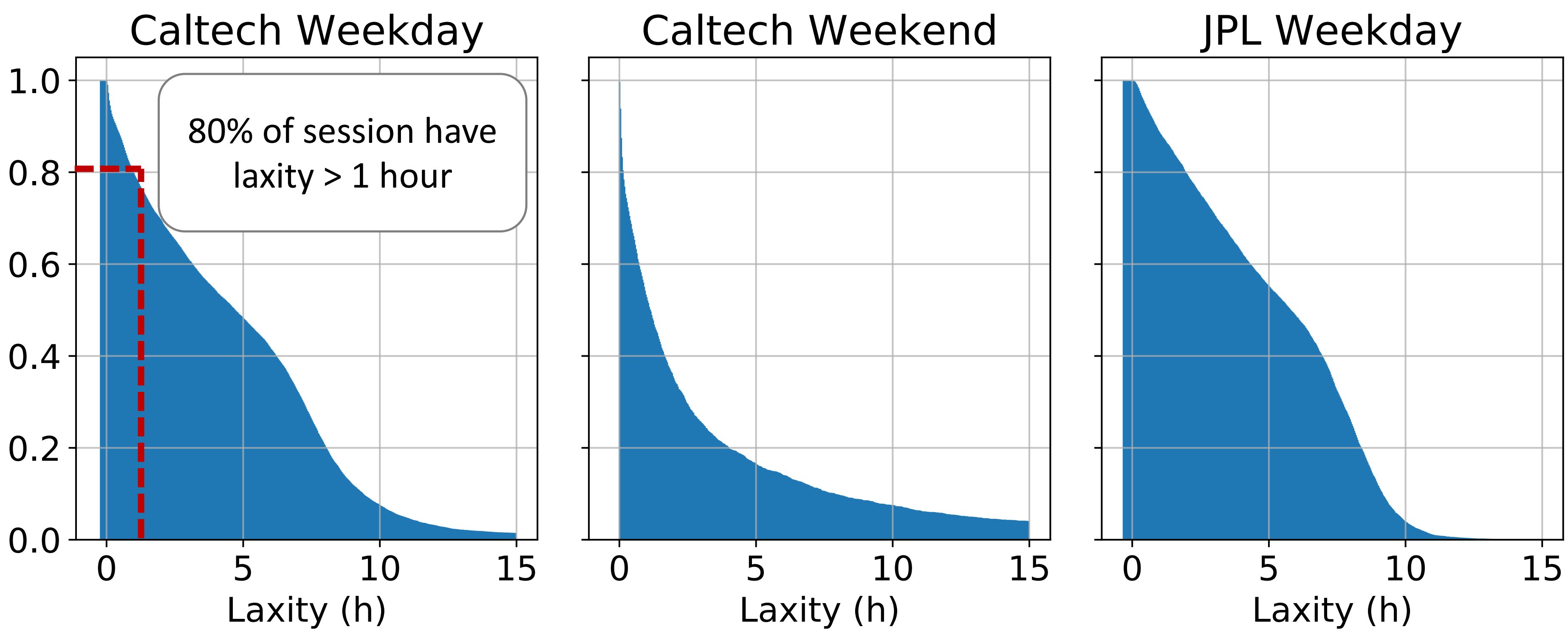
- 80,000+ EV charging sessions (March 2021)
- Publicly available: ev.caltech.edu
- Growing daily

Real fine-grained data for

- Modeling user behavior
- Evaluating charging algorithms
- Evaluating charging facilities
- Evaluating grid impacts

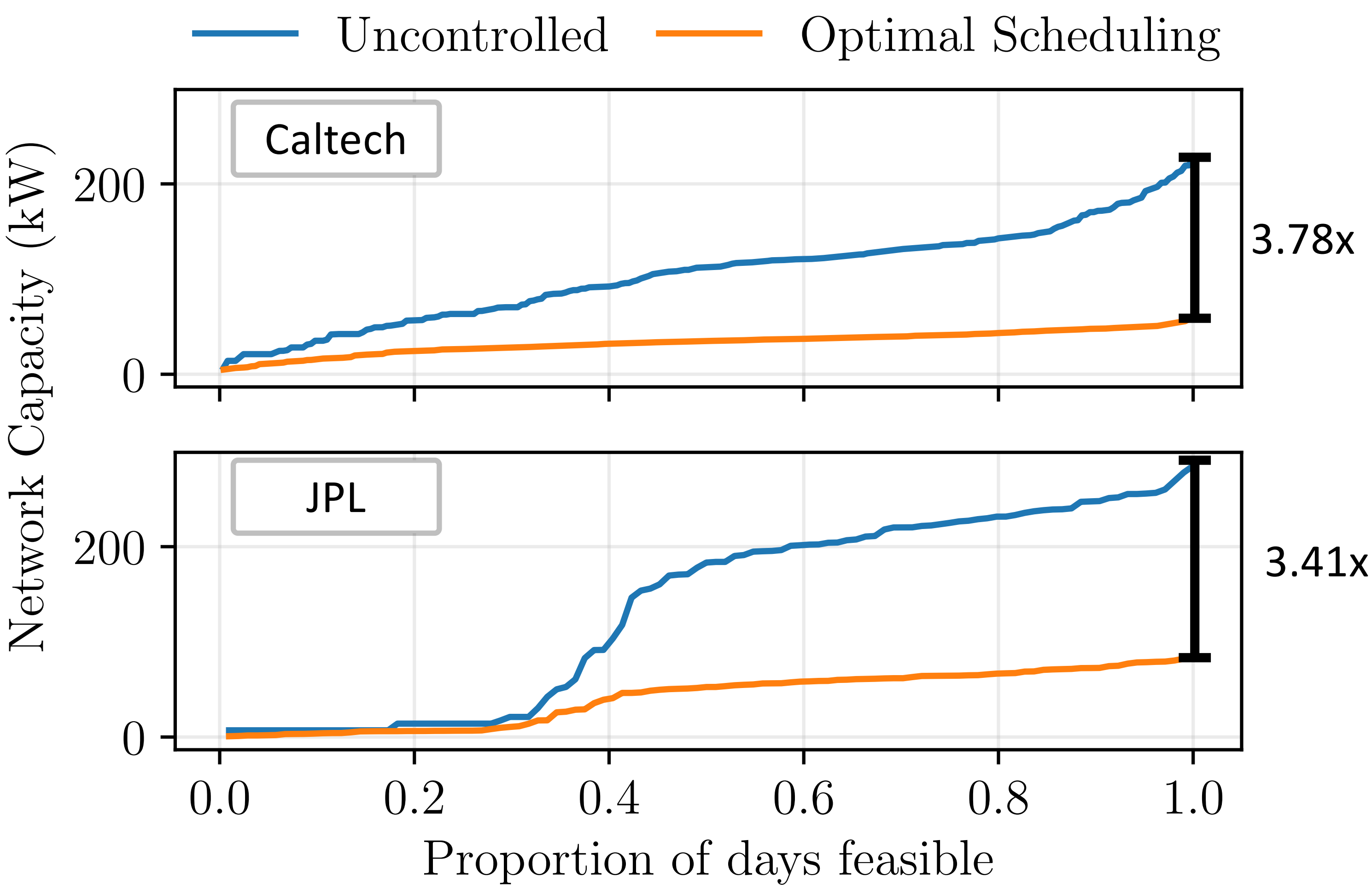
User flexibility

User flexibility



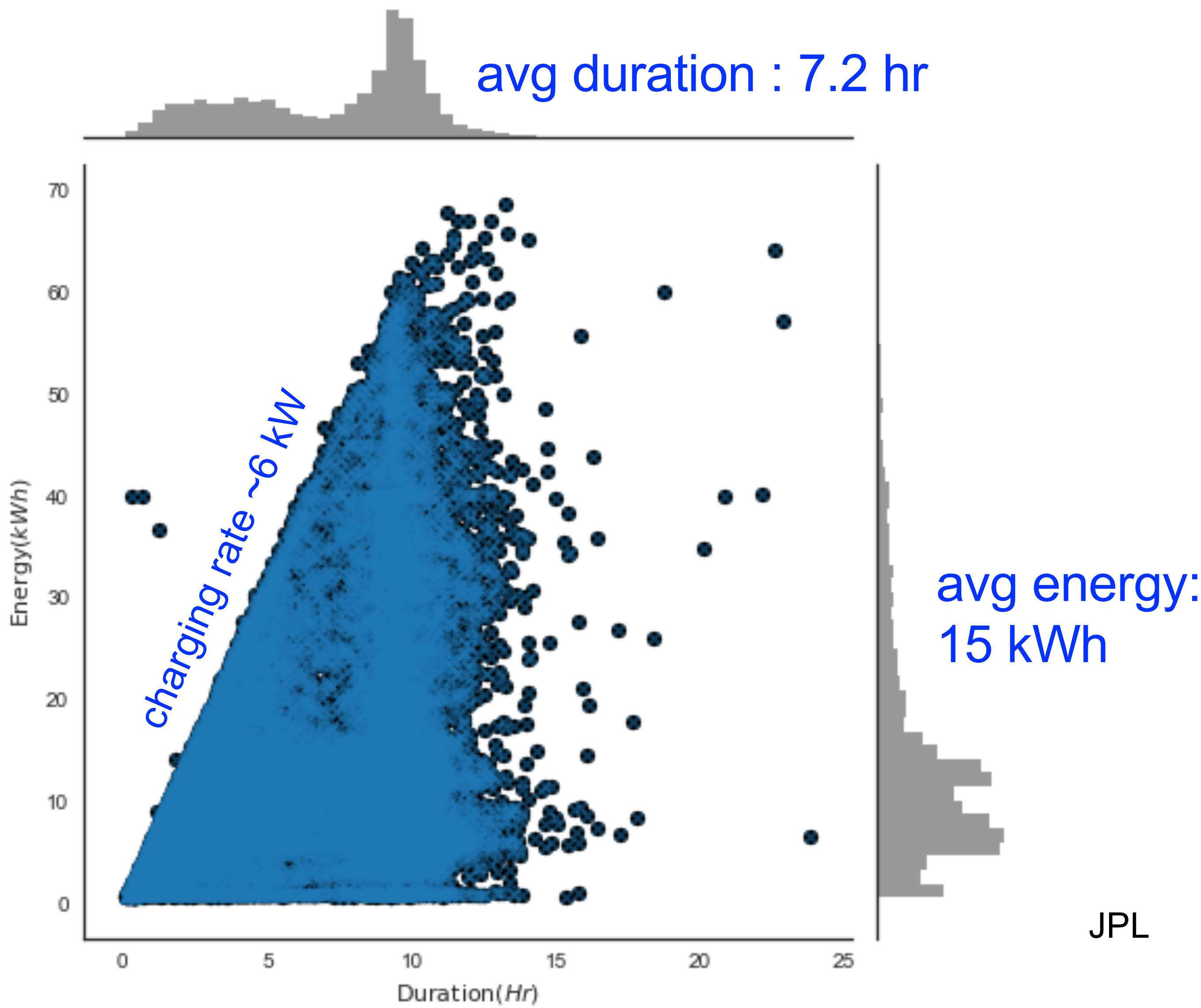
laxity := session duration - min charging time

ACN flexibility

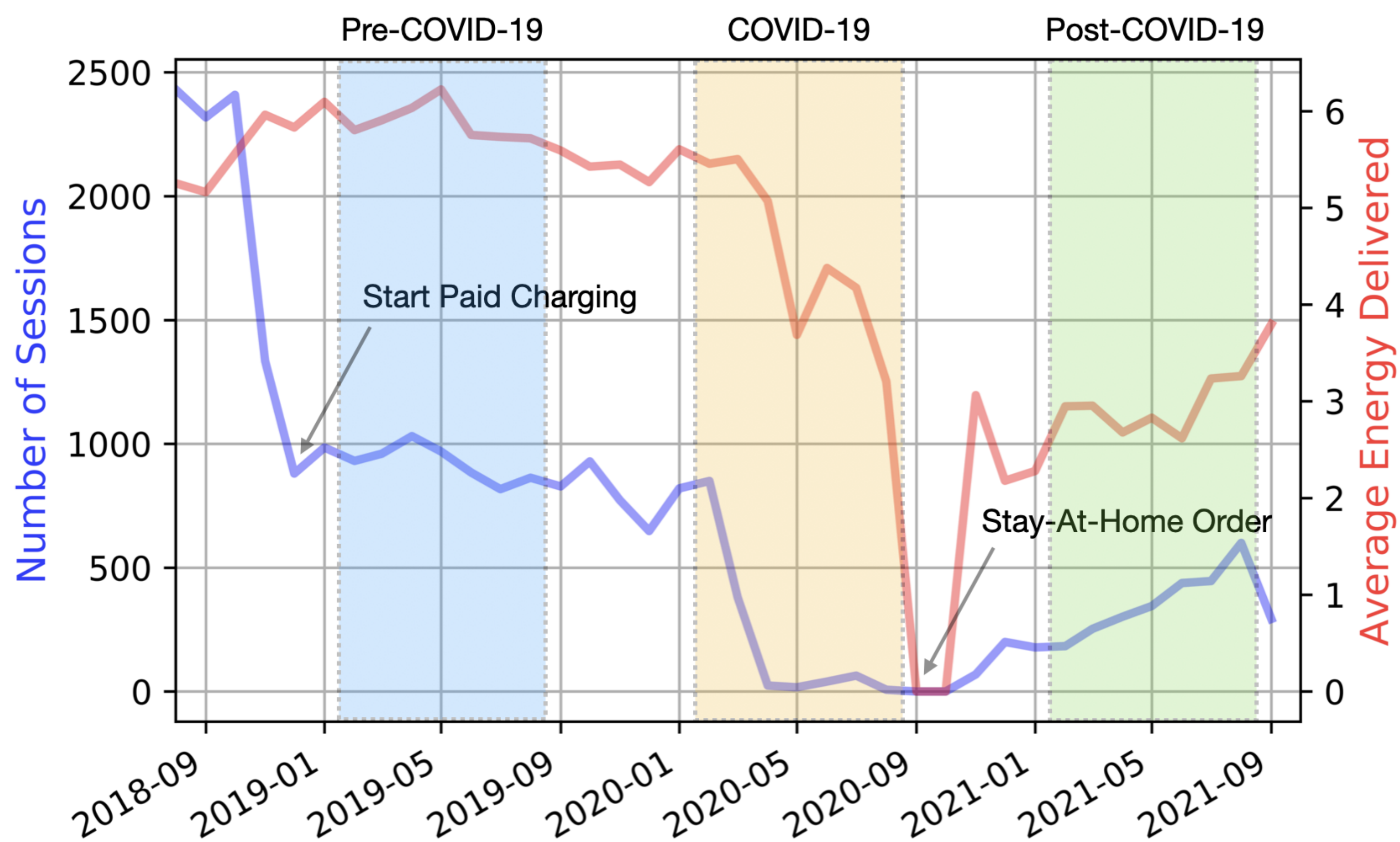


User behavior

Duration and energy delivered

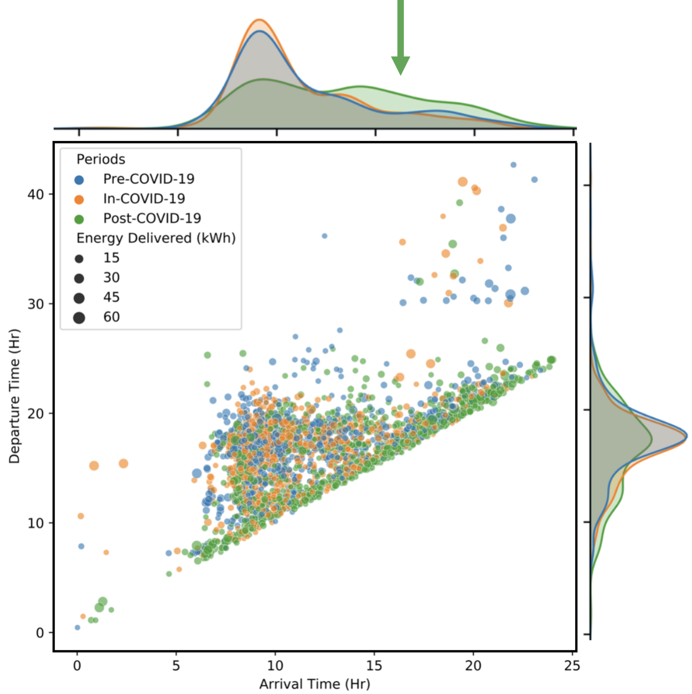


User behavior



effect of payment & COVID

arrival times have larger var post-COVID (more flex hours)

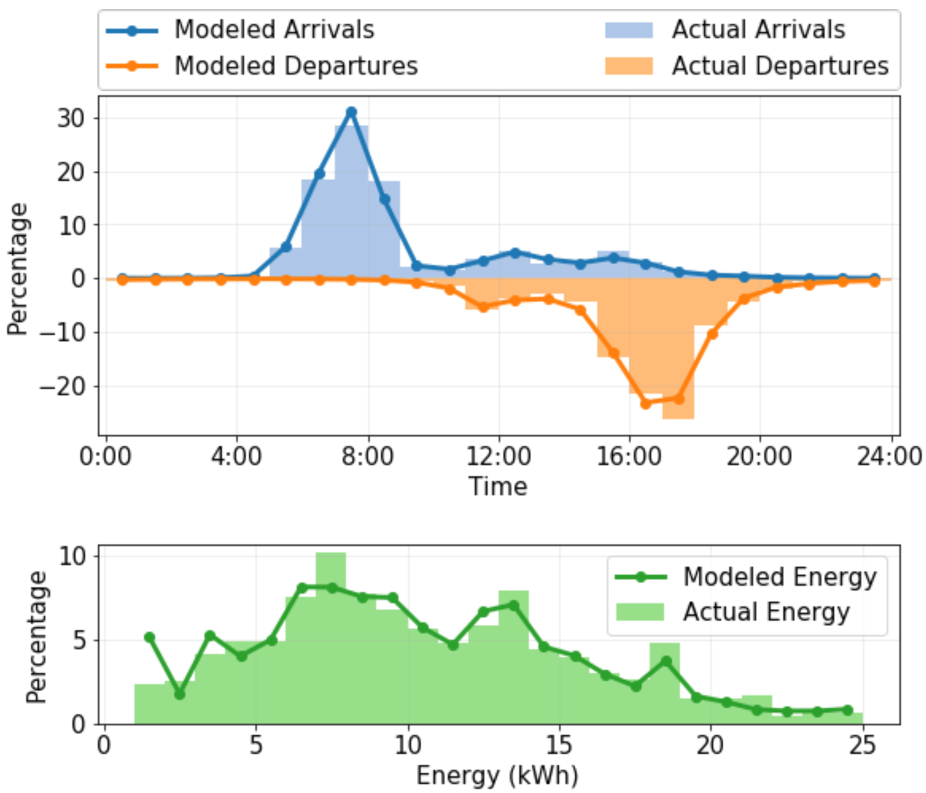


Caltech

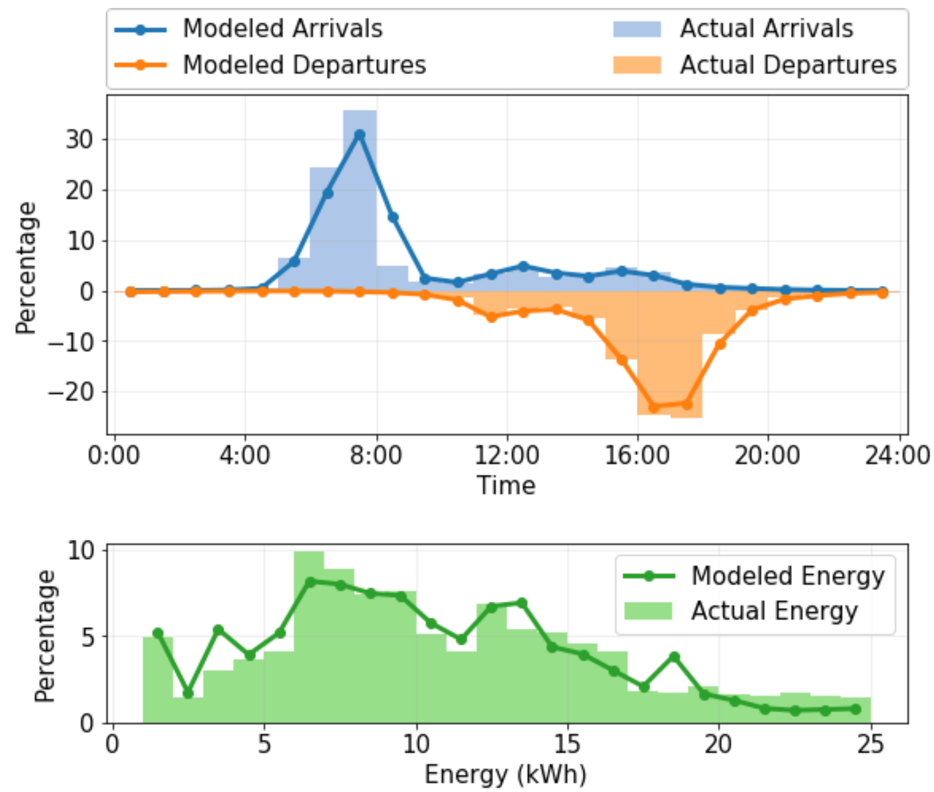
Learning user behavior

Gaussian mixture model

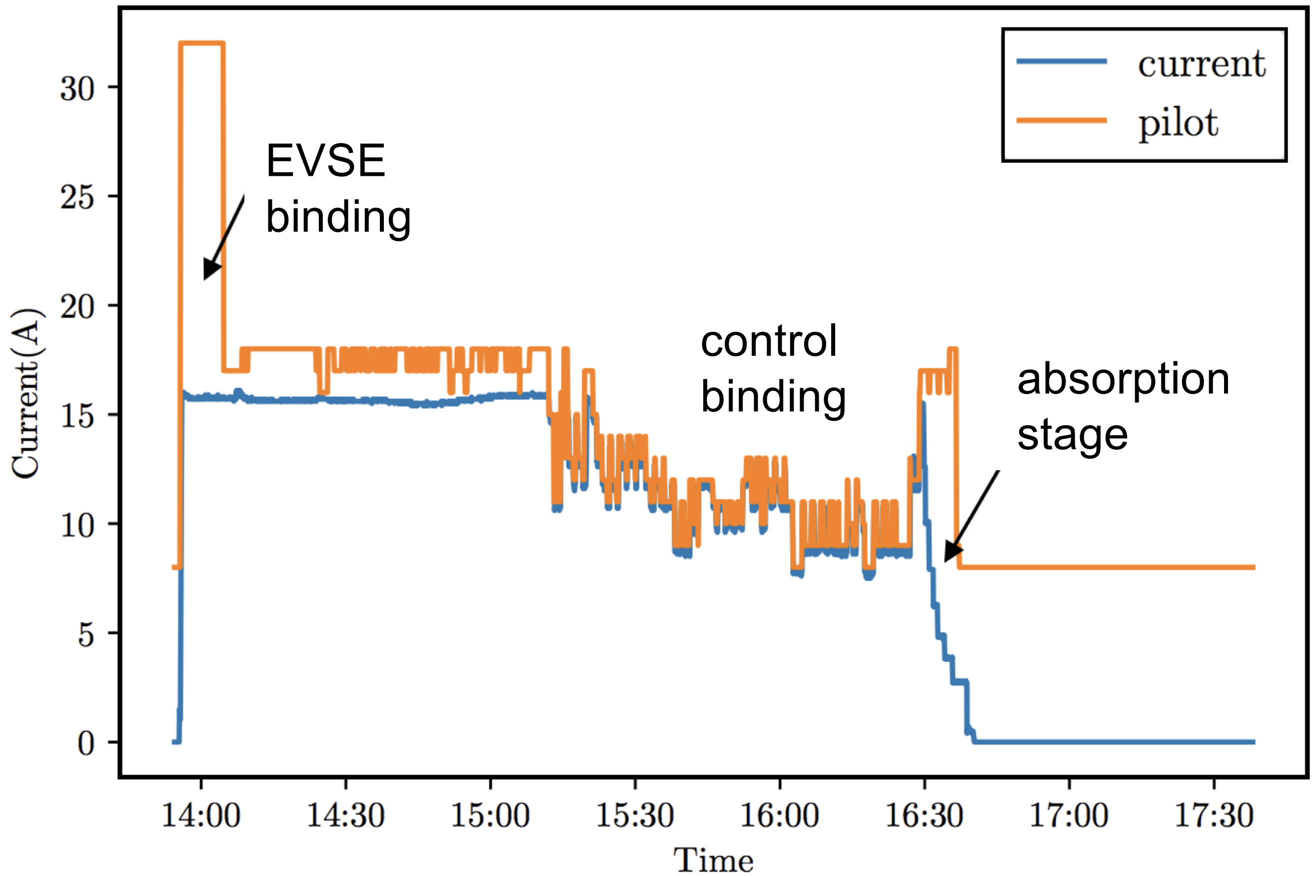
Testing Accuracy (9/1/18 – 11/1/18)



Evaluation Set Accuracy (12/1/18 – 5/1/19)



Charging curves



Caltech Oct 13, 2018

Time series: every 5-10 secs

- pilot signal from controller
- actual current drawn by EV

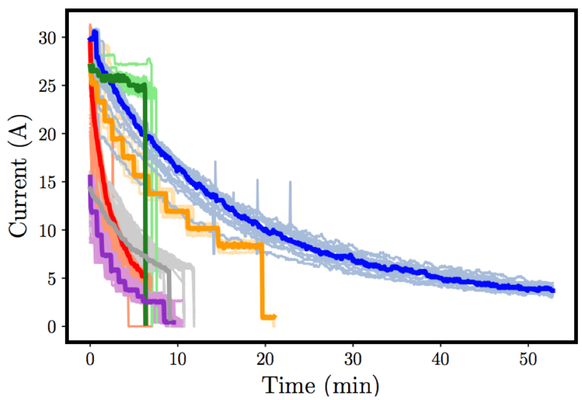
Learning charging curves

Goal: learn representative battery behaviors

- Only small # of batteries used by small # drivers underlying 35,000 charging curves

Challenge: do not know SoC

- Can only characterize tail behavior (absorption stage)
- Charging optimization, BMS actions, missing & noisy data

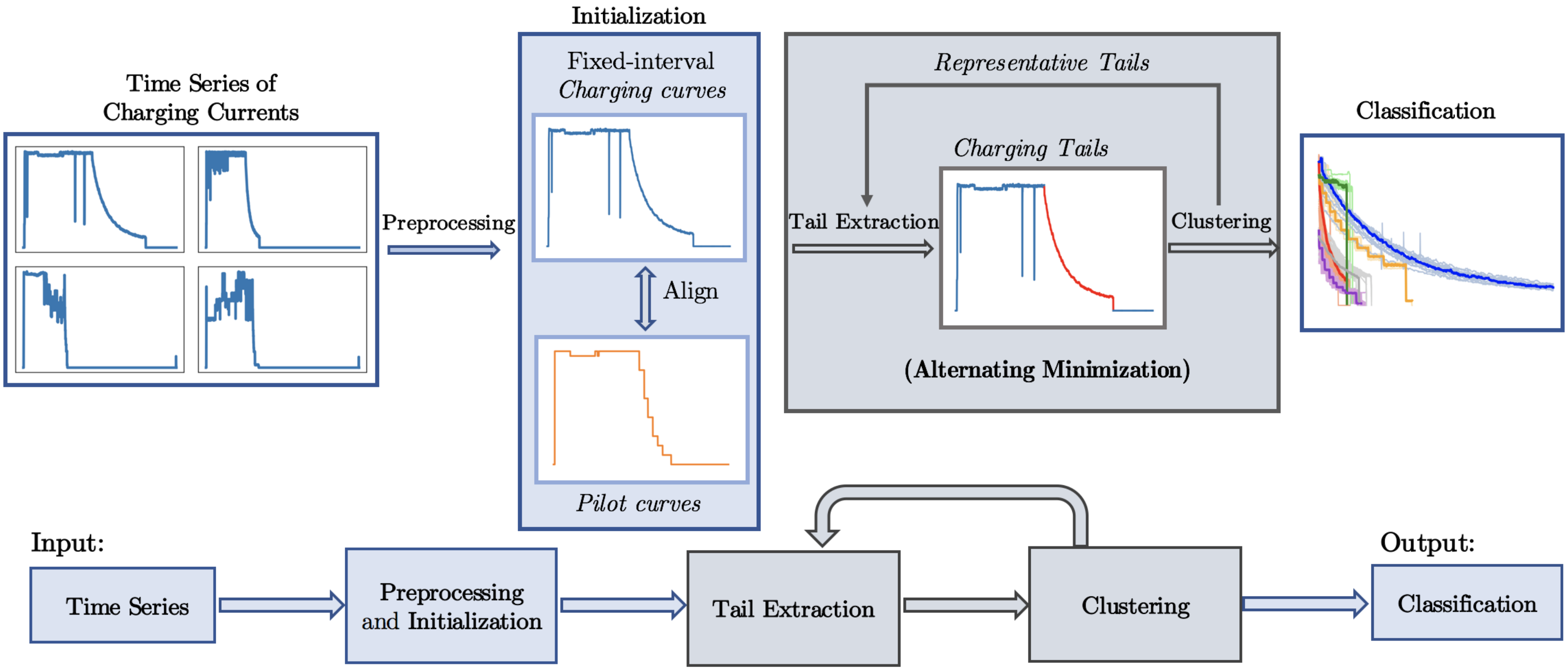


need to

- extract charging tails
- cluster charging tails

Chenxi Sun, Tongxin Li, S. H. Low and Victor Li.
Classification of EV charging time series with selective clustering
PSCC July 2020

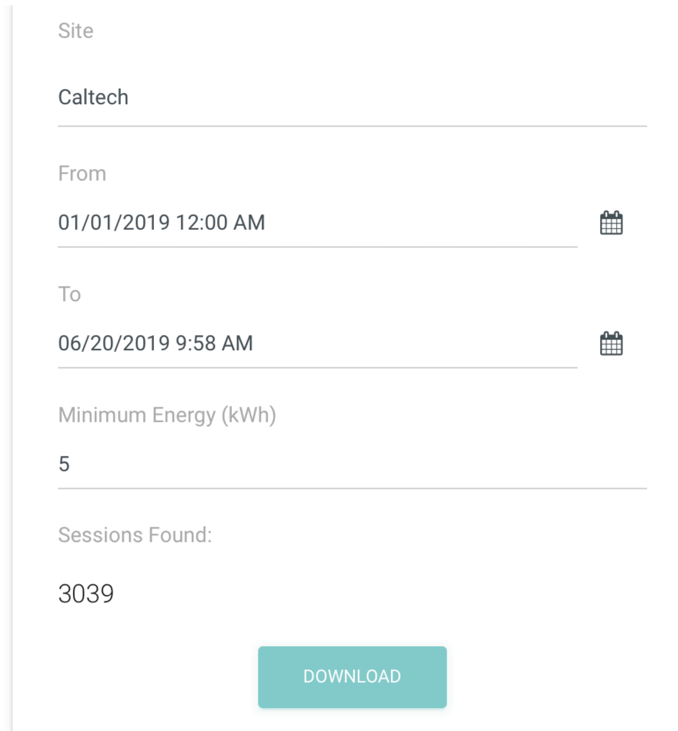
Learning charging curves



Chenxi Sun, Tongxin Li, S. H. Low and Victor Li.
Classification of EV charging time series with selective clustering
PSCC July 2020

Accessing ACN - Data

- Web Interface
- API
- Python Client
- ACN-Sim

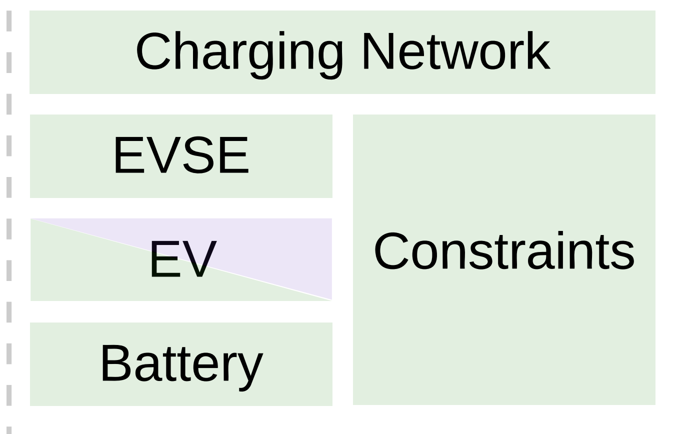


ev.caltech.edu

Caltech

open-source & extensible

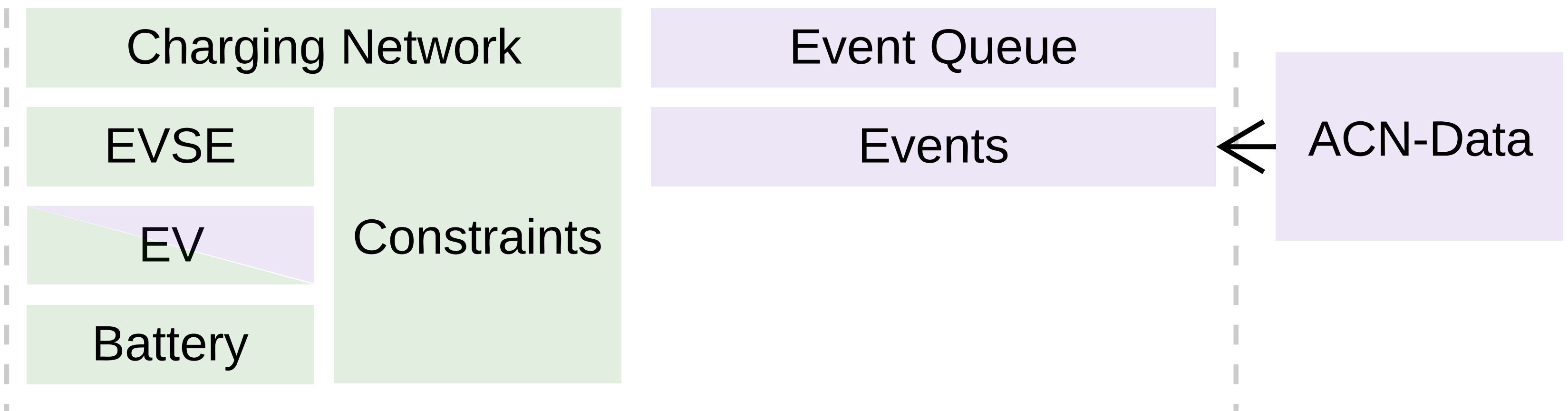
ACN - Sim



physical system /
simulation models

open-source & extensible

ACN - Sim

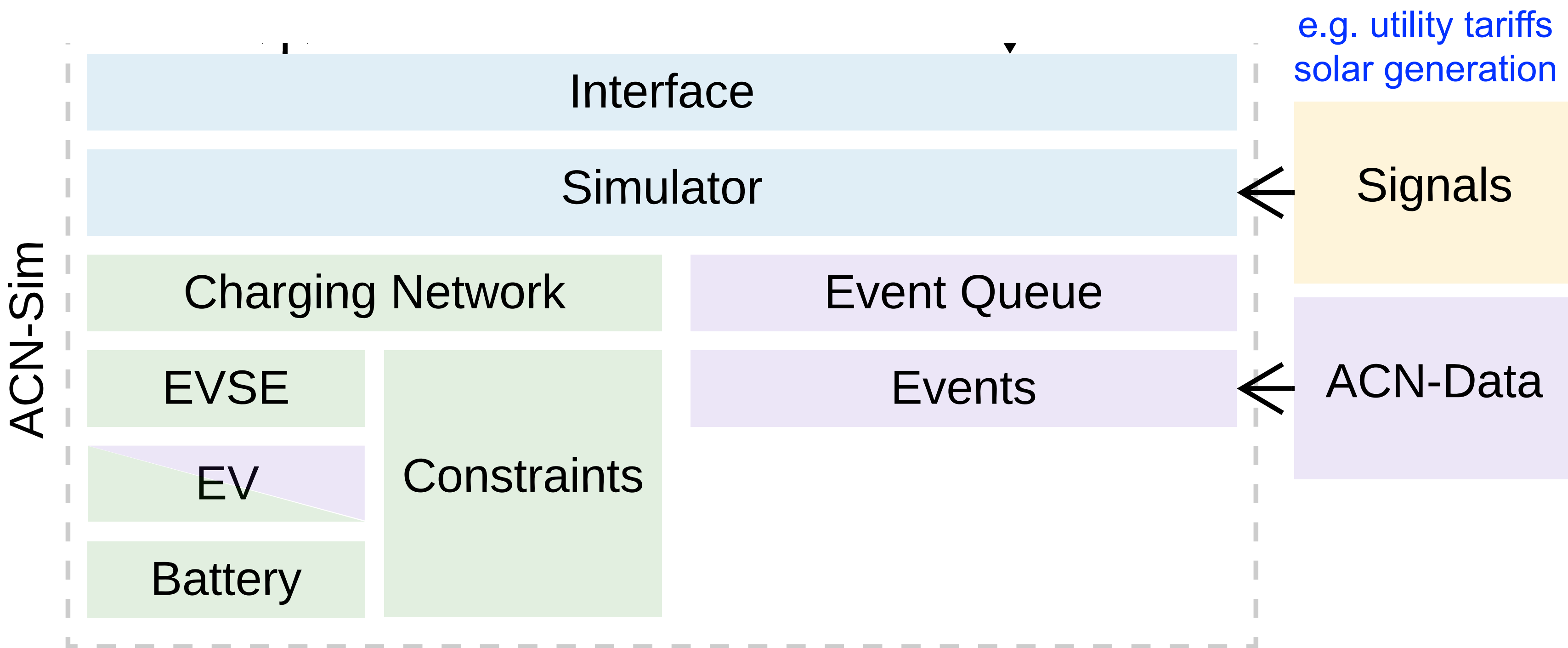


physical system / simulation models

integrated with ACN-Data

open-source & extensible

ACN - Sim

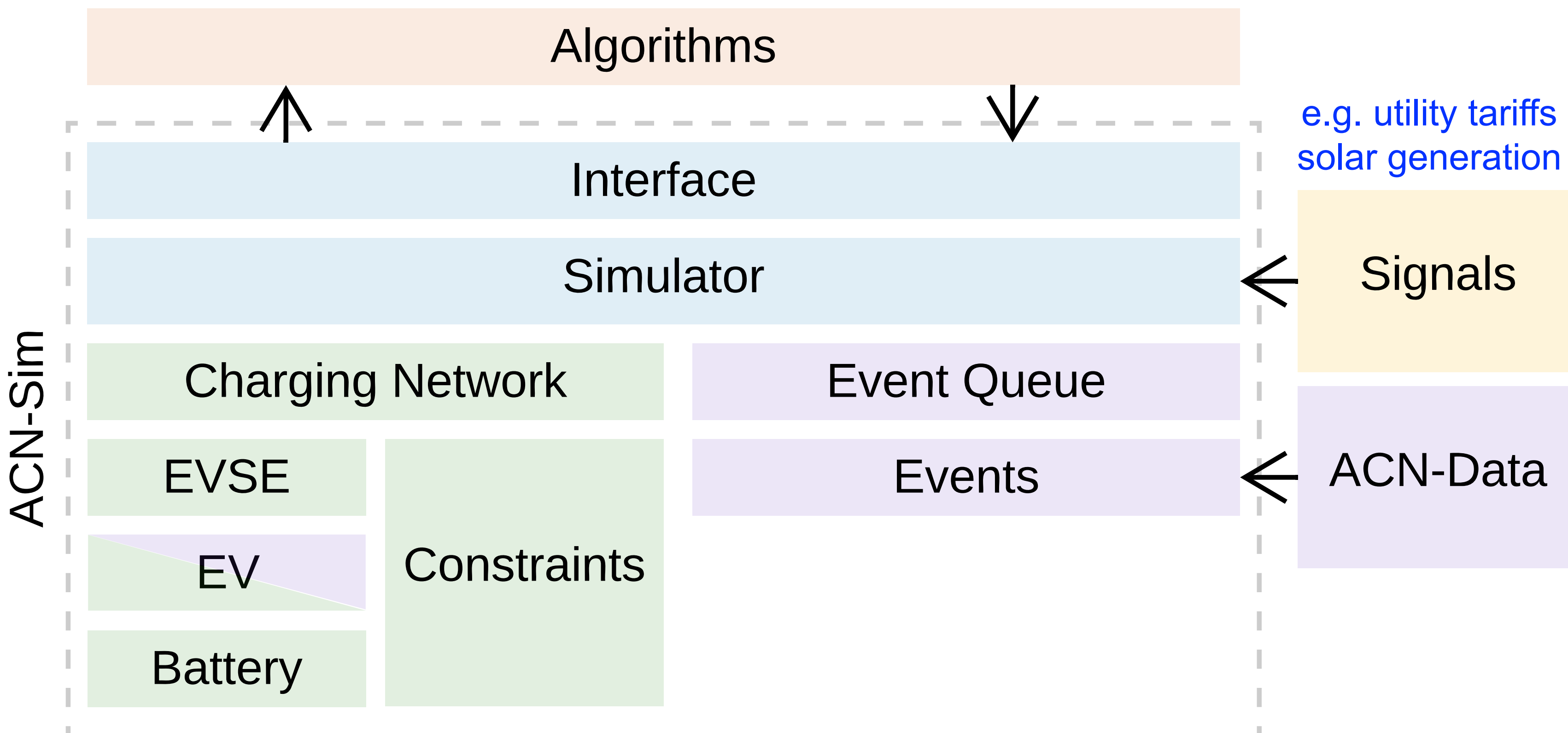


physical system /
simulation models

integrated with
ACN-Data

open-source & extensible

ACN - Sim



physical system / simulation models

integrated with ACN-Data

open-source & extensible

Grid impact

How can large-scale EV charging mitigate Duck Curve ?

Charging model

N EVs: $i = 1, \ldots, N$

T control intervals: $t = 1, \ldots, T$

EV i: $(e_i, a_i, d_i, \bar{r}_i)$

energy demand (miles / kWh)

arrival / departure time

peak charging rate (kW)

customizable utility functions

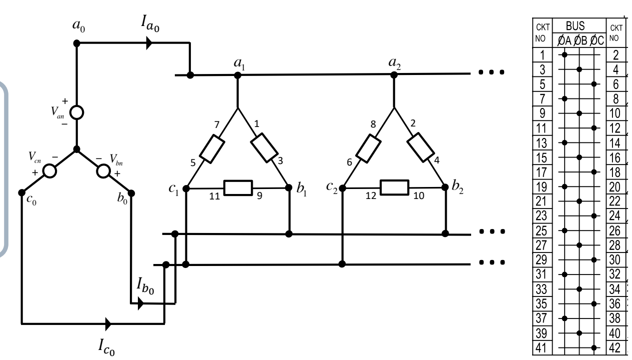
$$\max_r \sum_v \alpha_v u_v(r)$$

Compute: charging rates

$r := (r_i(t),\ i = 1, \ldots, N,\ t = 1, \ldots, T)$

$$0 \le r_i(t) \le \bar{r}_i(t)$$

$$\sum_{t \in \mathcal{T}} r_i(t) \le e_i$$



infrastructure constraints

$$\left| \sum_{i \in \mathcal{V}} A_{li} r_i(t) e^{j\phi_i} \right| \le c_{lt}(t)$$

SoC constraints, or linear approx.

Grid impact

MPC

$$\max_r \quad \sum_v \alpha_v u_v(r)$$

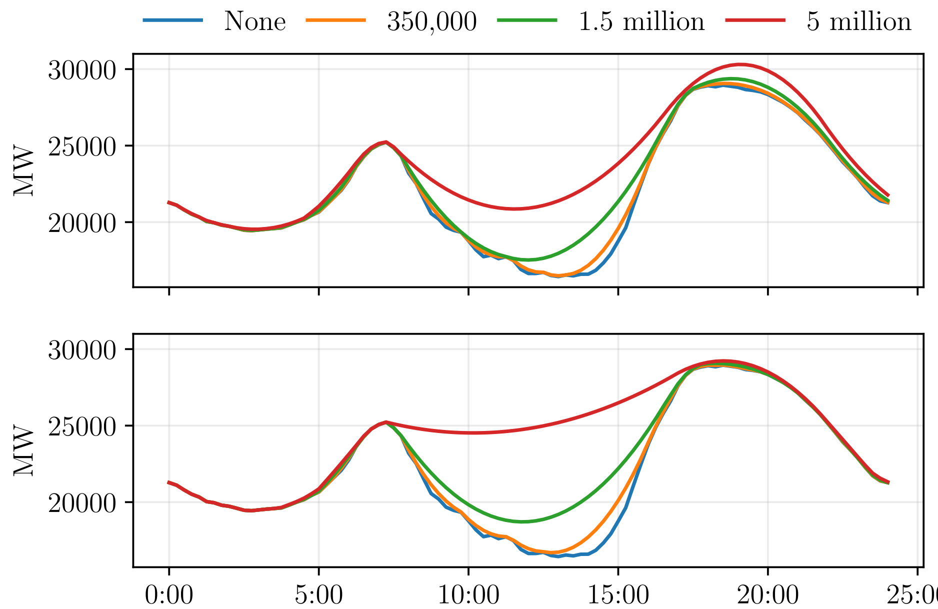
subject to

$$0 \leq r_i(t) \leq \bar{r}_i(t)$$

$$\sum_{t \in \mathcal{T}} r_i(t) \leq e_i$$

$$\left| \sum_{i \in \mathcal{V}} A_{li} r_i(t) e^{j\phi_i} \right| \leq c_{lt}(t)$$

MPC in real system is a lot more



Minimize evening ramp based on real data

- EV data from ACN-Data
- Simulation models from ACN-Sim
- CAISO solar and load data
- Simple estimate without grid model

ACN - Live

Algorithms

Interface

Safety / Reliability Checks

PowerFlex ACN Control System

open-source
& extensible

ACN research portal

Agenda

ACN: Caltech testbed

- Testbed to commercial deployment

ACN: Research Portal

- Data, Sim, Live

ACN: pricing demand charge

- Monthly billing at workplaces

Unbalanced 3-phase modeling

- Motivation, 3-phase network models

ACN Pricing

ELSEVIER

Electric Power Systems Research

Volume 189, December 2020, 106694

Pricing EV charging service with demand charge ☆

Zachary J. Lee [a], John Z.F. Pang [b], Steven H. Low [a, b]

PSCC 2020

Online adaptive charging

Model predictive control:

$$\max_{r} \quad \sum_{v} \alpha_v u_v(r)$$

$$\text{subject to} \quad 0 \;\leq\; r_i(t) \;\leq\; \bar{r}_i(t)$$

$$\sum_{t \in \mathcal{T}} r_i(t) \;\leq\; e_i$$

$$\left| \sum_{i \in \mathcal{V}} A_{li} r_i(t) e^{j\phi_i} \right| \;\leq\; c_{lt}(t)$$

Pricing design

Charging design

- Must adapt to system state in real time
- Objectives must be customized for site hosts

Pricing design: recover cost for site hosts

- Energy
- Externality: system peak (demand charge)
- Externality: infrastructure congestion

Key idea: decouple charging and pricing

- Drivers receive energy in time, at minimum payments
- Charging is socially optimized by MPC
- Site host fully recovers electricity cost

Offline optimal pricing

start with conclusion …

At end of month

- Compute ex post session price α_i^*
- Driver pays: $\sum_i \alpha_i^* e_i$

sum over driver's sessions

energy delivered in session i

No uncertainty nor need for ToU tariff or demand forecasts

Pricing design

$$C(r) \;:=\; \sum_t p_t \sum_i r_i(t) \;+\; P \max_t \sum_i r_i(t)$$

- p_t: time-varying tariff $/kWh
- P: demand charge $/kWh
- $\max_t \sum_i r_i(t)$: peak power

1. What is min system electricity cost to meet demand ?
2. How to fairly allocate system cost to drivers ?

Pricing design

$$C(r) \;:=\; \sum_t p_t \sum_i r_i(t) \;+\; P \max_t \sum_i r_i(t)$$

Pricing min system cost:

$$C^{\min} \;:=\; \min \;\; \sum_t p_t \sum_i r_i(t) \;+\; Pq$$

$$\text{s. t.} \;\; \sum_t r_i(t) \;=\; e_i,$$ meet demand α_i

$$\sum_i A_{li}\, r_i(t) \;\le\; c_{lt}$$ infrastructure capacity limit β_{lt}

$$r_i(t) \le \bar{r}_i(t),$$ EVSE limit γ_{it}

$$q \;\ge\; \sum_i r_i(t),$$ system peak δ_t

Pricing design

Fairly (incentive compatibly) allocate system cost to EVs

$$\pi_i^*(t) \quad := \quad \underbrace{p_t}_{\text{energy}} +$$

time-varying
tariff

Pricing design

Fairly (incentive compatibly) allocate system cost to EVs

$$\pi_i^*(t) \;:=\; \underbrace{p_t}_{\text{energy}} + \underbrace{\sum_l A_{li}\beta_{lt}^*}_{\text{network congestion}} + \underbrace{\gamma_{it}^*}_{\substack{\text{charger}\\\text{congestion}}} + \underbrace{\delta_t^*}_{\text{demand charge}}$$

- Driver & time dependent prices

Driver pays for each session i

$$\Pi_i^* \;:=\; \sum_t \pi_i^*(t)\, r_i^*(t)$$

This achieves pricing goals: recovers

- Energy cost
- Congestion rents
- Demand charge EV i is responsible for

Pricing design

Design principle:

$$\pi_i^*(t) \quad := \quad \underbrace{p_t}_{\text{energy}} + \underbrace{\sum_l A_{li}\beta_{lt}^*}_{\text{network congestion}} + \underbrace{\gamma_{it}^*}_{\substack{\text{charger}\\ \text{congestion}}} + \underbrace{\delta_t^*}_{\text{demand charge}}$$

$$\Pi_i^* \quad = \quad \sum_t \pi_i^*(t) r_i^*(t)$$

Theorem

1. Demand charge: $P = \sum_t \delta_t^*$ EVs that cause peak will pay

Pricing design

Design principle:

$$\pi_i^*(t) \;:=\; \underbrace{p_t}_{\text{energy}} + \underbrace{\sum_l A_{li}\beta_{lt}^*}_{\text{network congestion}} + \underbrace{\gamma_{it}^*}_{\substack{\text{charger}\\\text{congestion}}} + \underbrace{\delta_t^*}_{\text{demand charge}}$$

$$\Pi_i^* \;=\; \sum_t \pi_i^*(t) r_i^*(t)$$

Theorem

1. Demand charge: $P = \sum_t \delta_t^*$ EVs that cause peak will pay

2. Time-invariant session price α_i^*: $\Pi_i^* = \alpha_i^* e_i$

$$\pi_i^*(t) \geq \alpha_i^* \quad \text{with} \quad \pi_i^*(t) = \alpha_i^* \;\; \text{if} \;\; r_i^*(t) > 0$$

EVs pay min cost

Pricing design

Design principle:

$$\pi_i^*(t) \quad := \quad \underbrace{p_t}_{\text{energy}} + \underbrace{\sum_l A_{li}\beta_{lt}^*}_{\text{network congestion}} \quad + \underbrace{\gamma_{it}^*}_{\substack{\text{charger}\\ \text{congestion}}} \quad + \underbrace{\delta_t^*}_{\text{demand charge}}$$

$$\Pi_i^* \quad = \quad \sum_t \pi_i^*(t) r_i^*(t)$$

<u>Theorem</u>

1. Demand charge: $P = \sum_t \delta_t^*$ EVs that cause peak will pay

2. Time-invariant session price α_i^*: $\Pi_i^* = \alpha_i^* \, e_i$

$$\pi_i^*(t) \geq \alpha_i^* \quad \text{with} \quad \pi_i^*(t) = \alpha_i^* \;\; \text{if} \;\; r_i^*(t) > 0$$

EVs pay min cost

3. Cost recovery: $\sum_i \Pi_i^* \quad \geq \quad C^{min}$

$$\sum_i \Pi_i^* - C^{min} \quad = \quad \sum_{t,l} c_{lt}\beta_{lt}^* + \sum_{t,i} \bar{r}_i(t)\gamma_{it}^*$$

Congestion rents

[Lee, Pang, Low. PSCC 2020]

Offline optimal pricing

At end of month

- Compute ex post session price α_i^*
- Driver pays: $\sum_i \alpha_i^* e_i$

No uncertainty nor need for ToU tariff or demand forecasts

Agenda

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Unbalanced 3-phase modeling

- Motivation, 3-phase network models

Unbalance 3-phase modeling

Power System Analysis

A Mathematical Approach

Steven H. Low

DRAFT available at: http://netlab.caltech.edu/book/

Corrections, questions, comments appreciated!

Motivation

Most papers implicitly assume single-phase

- Balanced 3-phase systems have single-phase equivalents

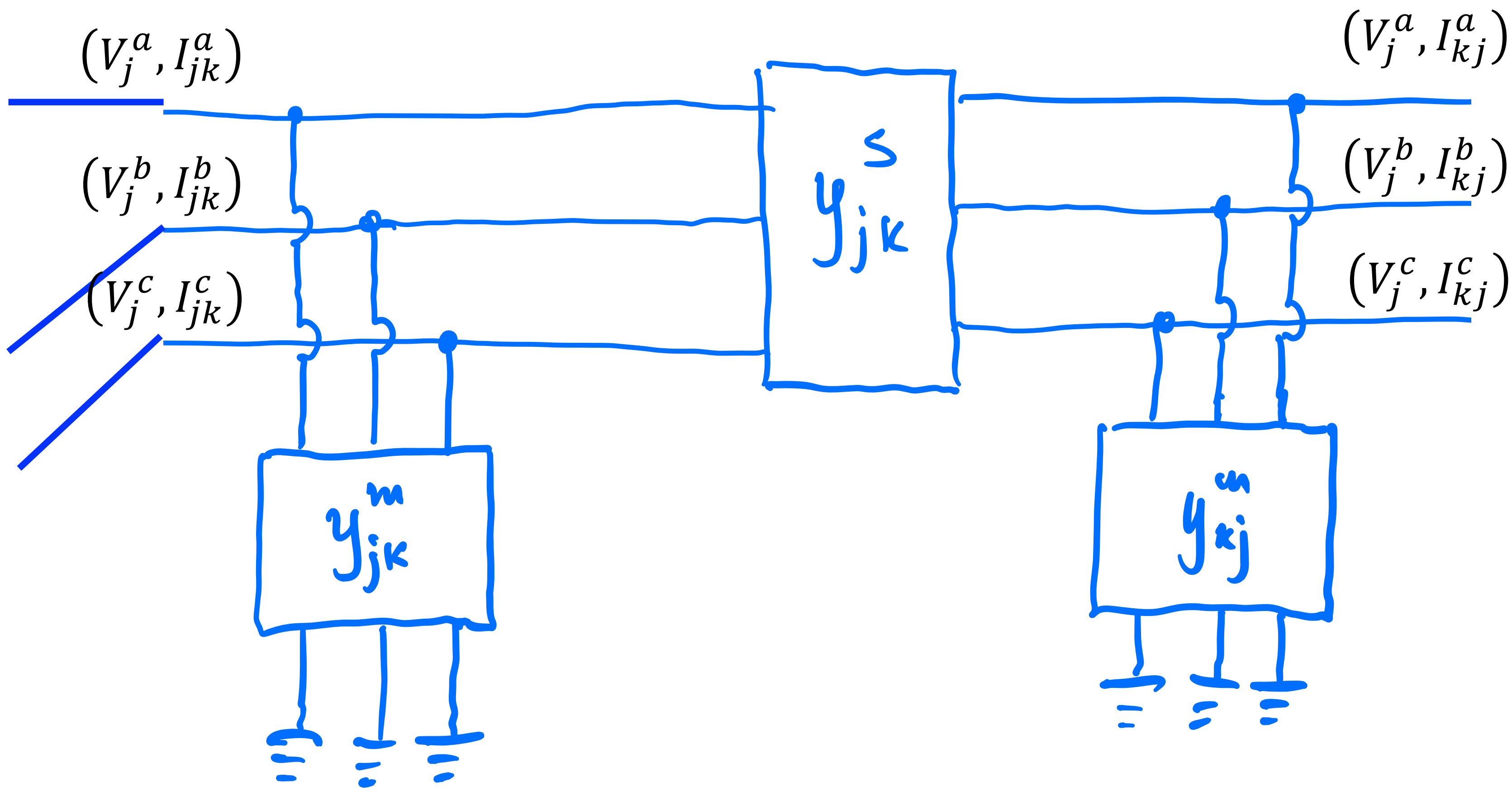
Single-phase models applicable for most purposes

- Transmission system applications
- For illustrating basic ideas and analysis of most algorithms (unbalanced 3-phase models structurally similar to 1-phase models)

Unbalanced 3-phase modeling needed

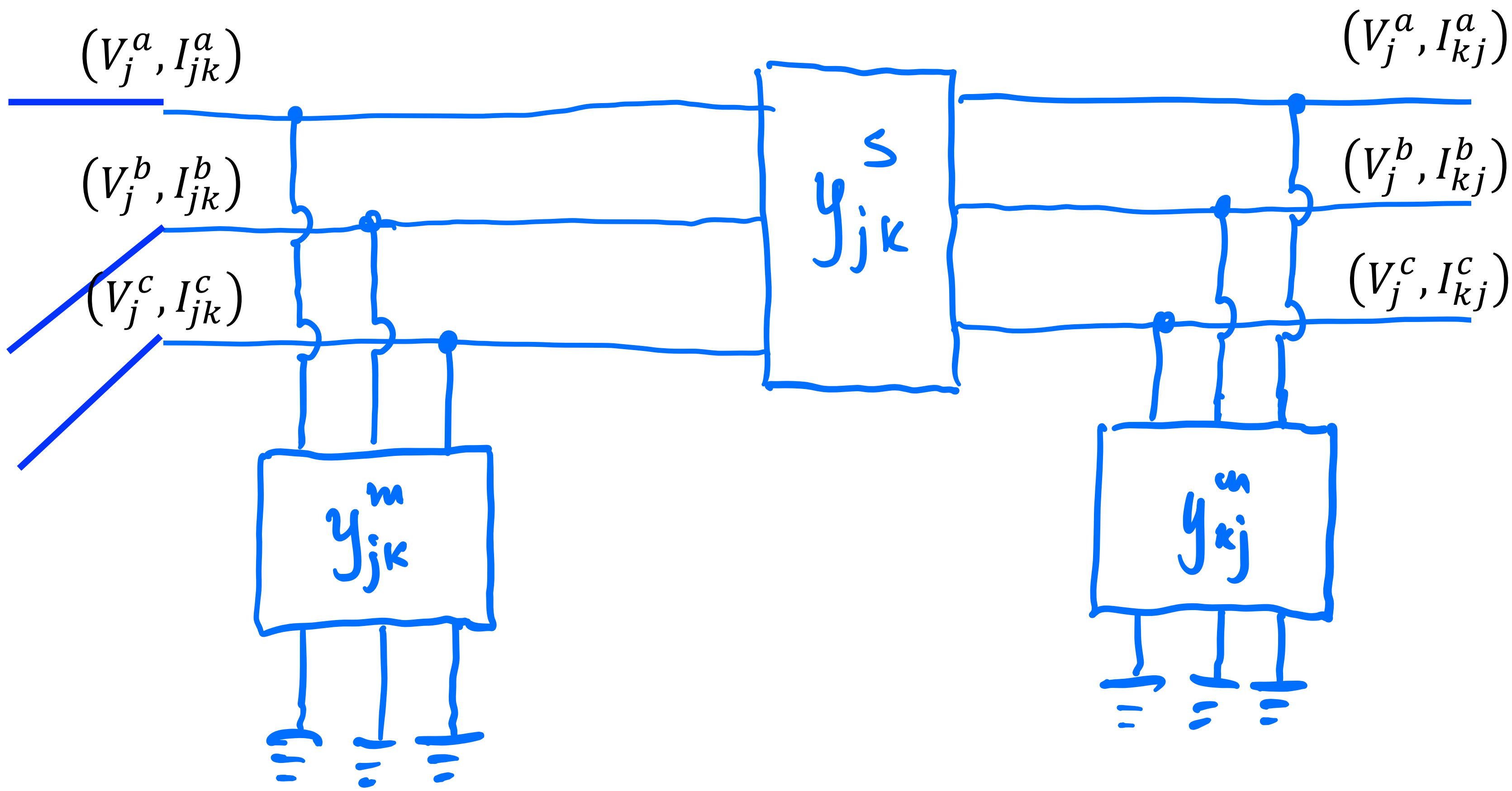
- When control & optimization are explicitly on single-phase devices making up a 3–phase devices
- For implementation in real systems when phases are not balanced

Motivation



- Many models assume terminal currents $(I_{jk}^a, I_{jk}^b, I_{jk}^c)$ are controllable (optimization vars)
- Extension to 3-phase setting is straightforward:

Motivation



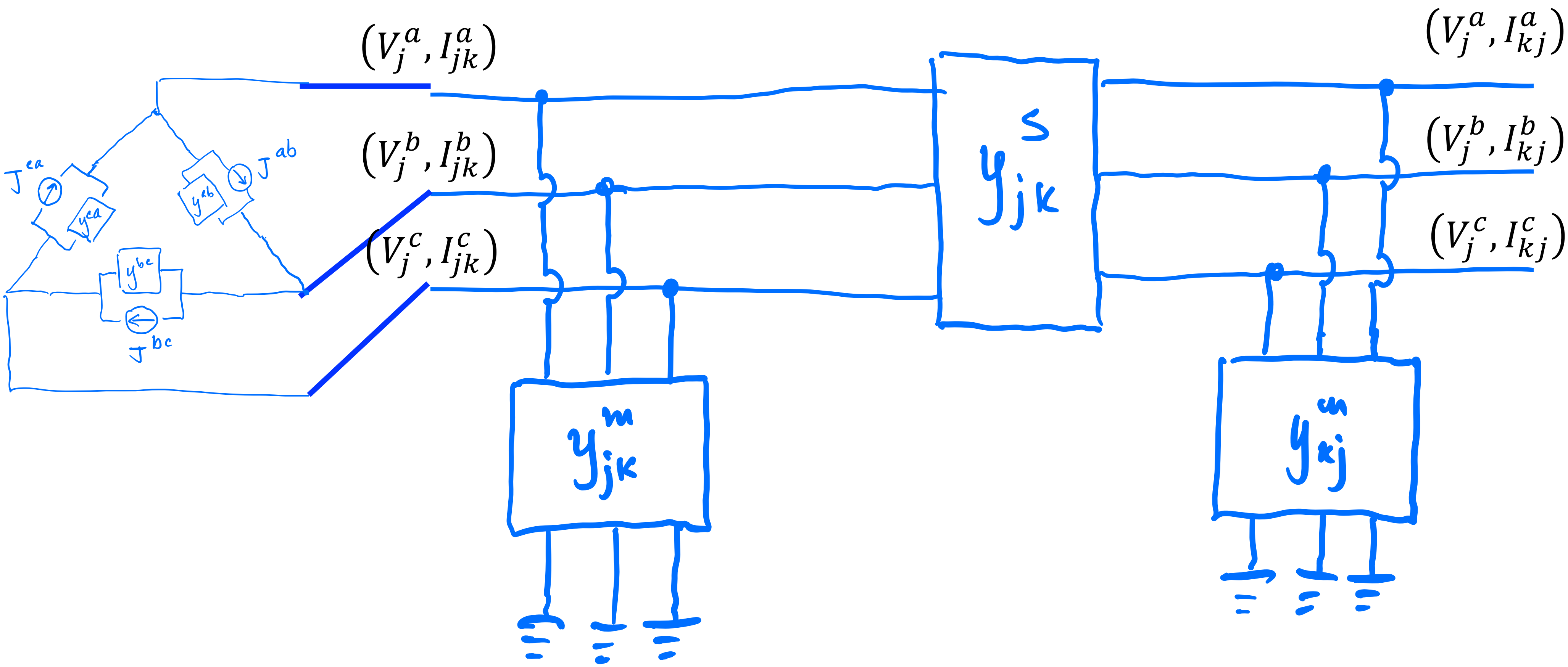
$$I_{jk} = y_{jk}^s \left(V_j - V_k \right) + y_{jk}^m V_j$$

$$I_{kj} = y_{jk}^s \left(V_k - V_j \right) + y_{kj}^m V_k$$

1-phase: $I_{jk}, V_j^a \in \mathbb{C}$. $y_{jk}^{s/m} \in \mathbb{C}$

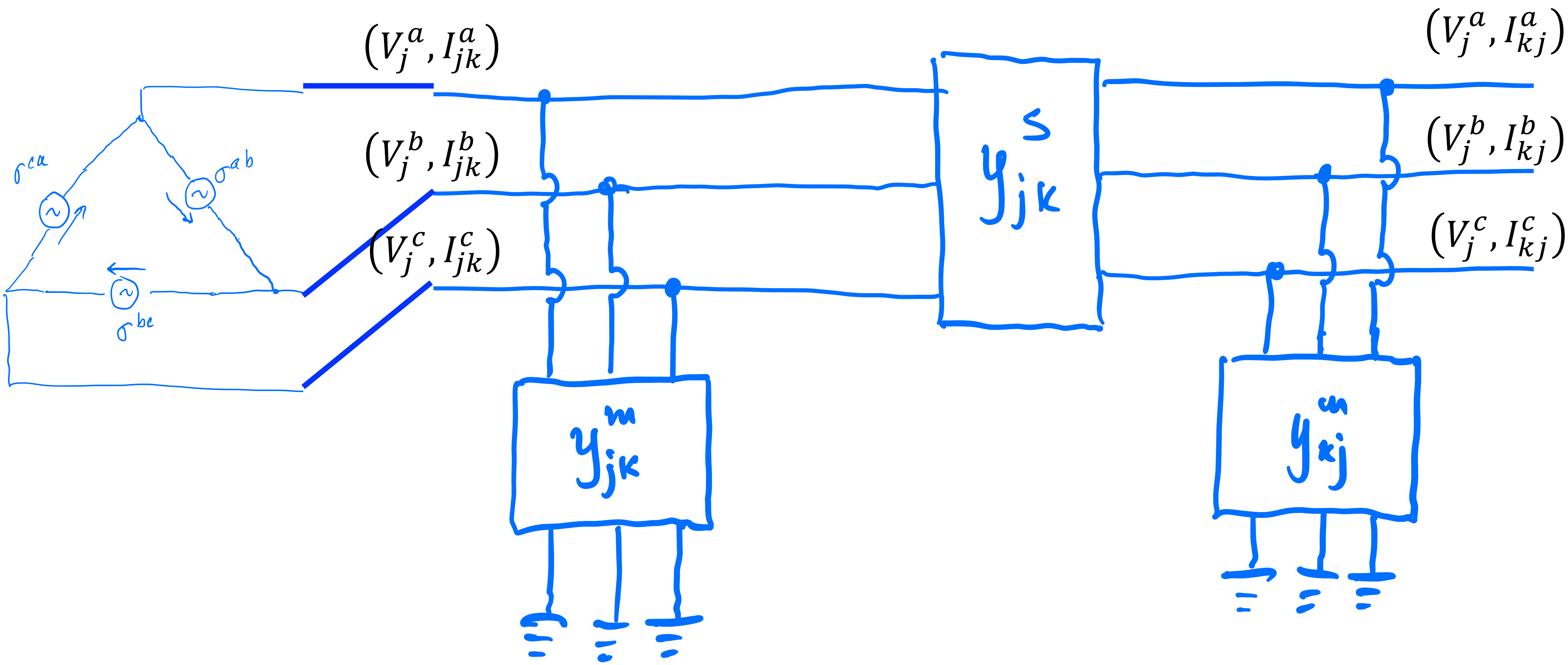
3-phase: $I_{jk}, V_j^a \in \mathbb{C}^3$. $y_{jk}^{s/m} \in \mathbb{C}^{3\times3}$

Motivation



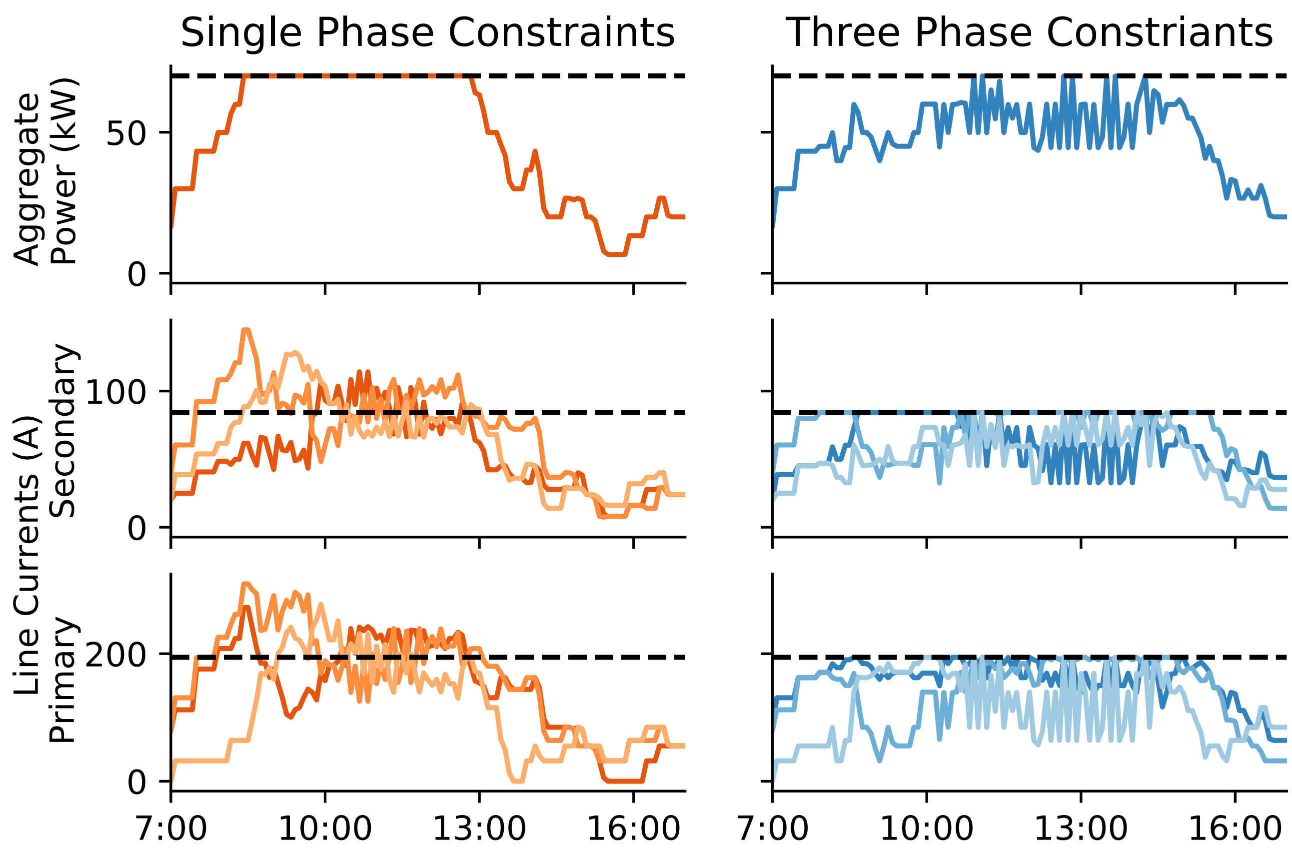
- Terminal currents I_{jk} are externally observable, but often not directly controllable
- If only internal currents $(J_j^{ab}, J_j^{bc}, J_j^{ca})$ of current source are directly controllable, then need a 3-phase device model to convert between internal & terminal vars

Motivation



Similarly for power sources or voltage sources

Motivation: example



SOC constraints

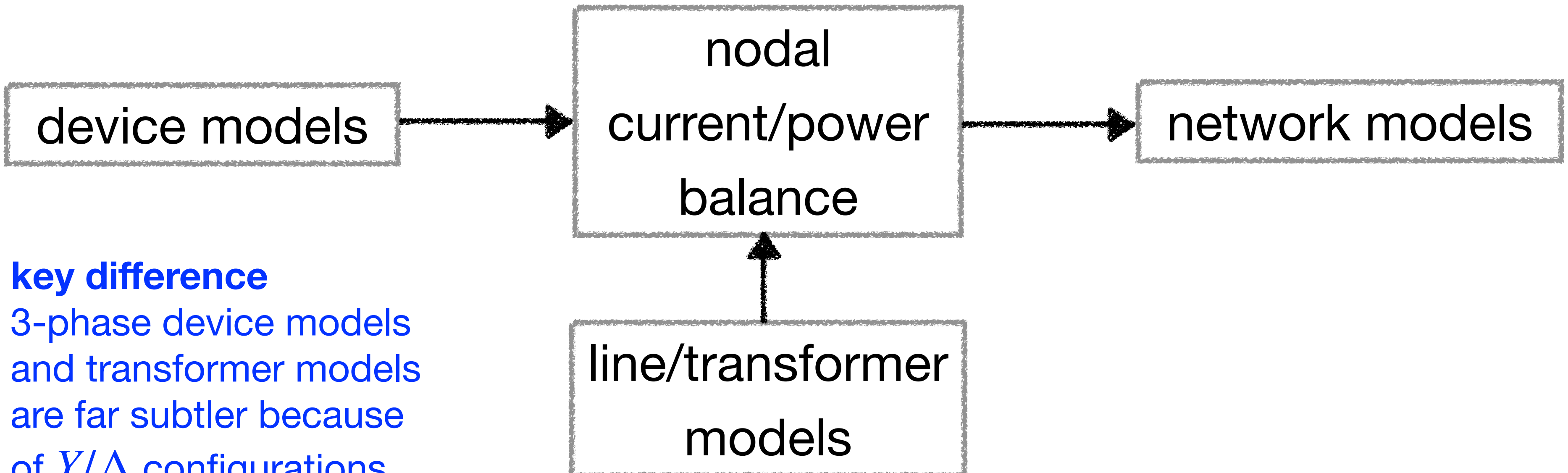
SOC constraints

Left panel: Actual 3-phase currents violate capacity constraints if “single-phase constraints” are used (ACN-Sim based on Caltech ACN on Sept 5, 2018 data)

“single-phase constraints” : $\sum_i r_i(t) \leq R$ (no phase line constraints for lack of phase info)

[Lee et al (2021), ACN-Sim, TSG]

Overview: 3-phase modeling

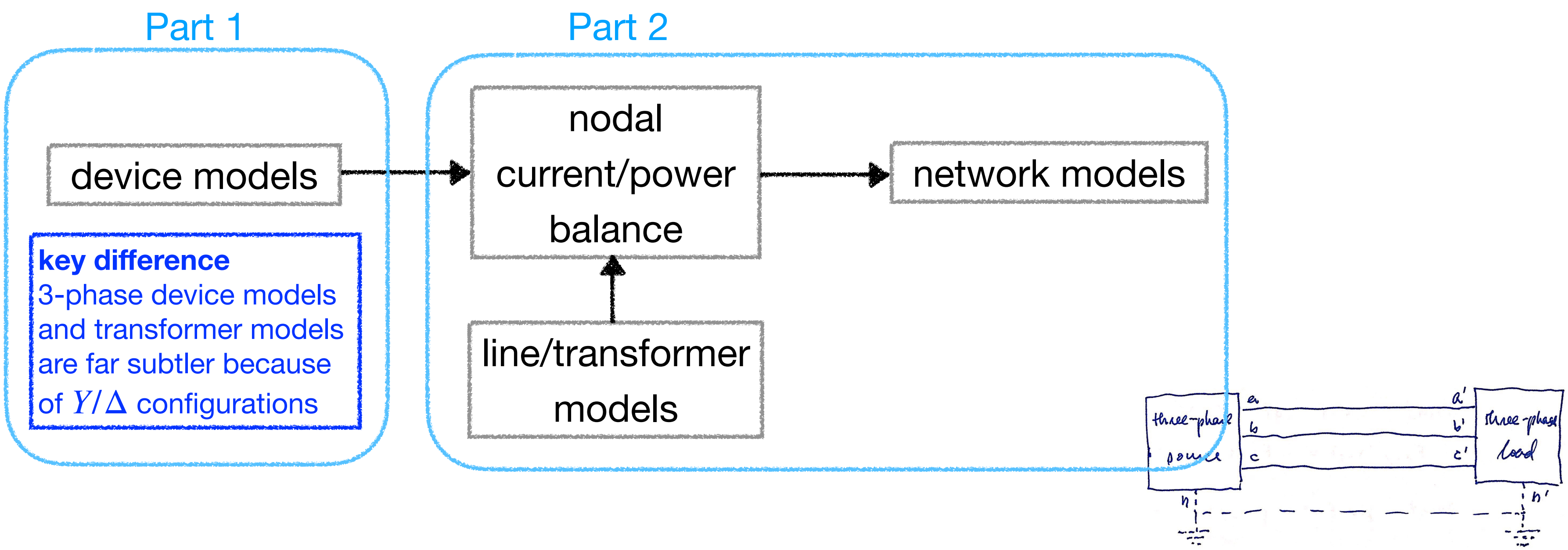


key difference
3-phase device models and transformer models are far subtler because of Y/Δ configurations

single-phase or 3-phase

3-phase Power Flow Model

Overview



single-phase or 3-phase

Key question

How to derive external models of 3-phase devices

1. Voltage/current/power sources, impedances (1-phase device: internal models)
2. … in Y/Δ configurations (conversion rules: int → ext)
3. … with or without neutral lines, grounded or ungrounded, zero or nonzero grounding impedances

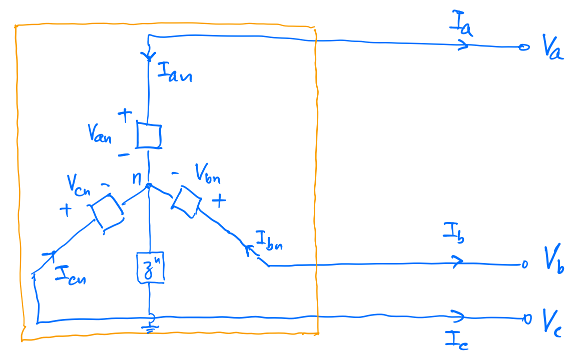
Propose a simple and unified method to derive external models

Internal variables

Y configuration

Internal voltage, current, power across single-phase devices:

$$V^Y := \begin{bmatrix} V^{an} \\ V^{bn} \\ V^{cn} \end{bmatrix}, \ I^Y := \begin{bmatrix} I^{an} \\ I^{bn} \\ I^{cn} \end{bmatrix}, \ s^Y := \begin{bmatrix} s^{an} \\ s^{bn} \\ s^{cn} \end{bmatrix} := \begin{bmatrix} V^{an}\bar{I}^{an} \\ V^{bn}\bar{I}^{bn} \\ V^{cn}\bar{I}^{cn} \end{bmatrix}$$



neutral voltage (wrt common reference pt) $V^n \in \mathbb{C}$

neutral current (away from neutral) $I^n \in \mathbb{C}$

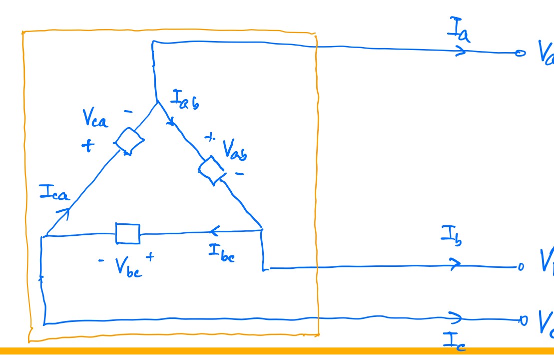
Device may or may not be grounded, and neutral impedance z^n may or may not be zero

Internal variables

Δ configuration

Internal voltage, current, power across single-phase devices:

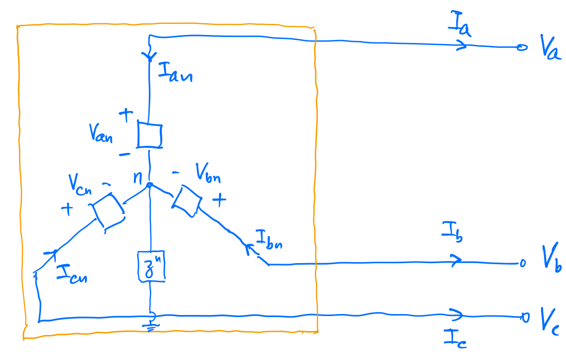
$$V^{\Delta} := \begin{bmatrix} V^{ab} \\ V^{bc} \\ V^{ca} \end{bmatrix}, \quad I^{\Delta} := \begin{bmatrix} I^{ab} \\ I^{bc} \\ I^{ca} \end{bmatrix}, \quad s^{\Delta} := \begin{bmatrix} s^{ab} \\ s^{bc} \\ s^{ca} \end{bmatrix} := \begin{bmatrix} V^{ab}\bar{I}^{ab} \\ V^{bc}\bar{I}^{bc} \\ V^{ca}\bar{I}^{ca} \end{bmatrix}$$



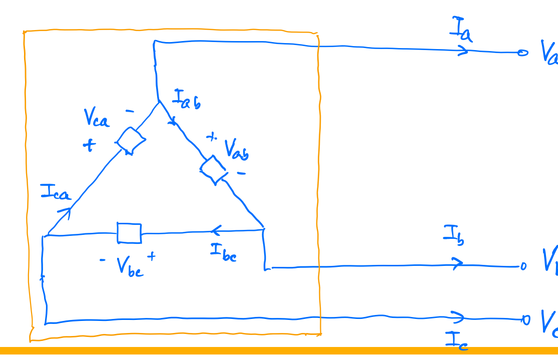
Terminal variables

Terminal voltage, current, power (for both Y and Δ) to reference:

$$V := \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}, \; I := \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}, \; s := \begin{bmatrix} s^a \\ s^b \\ s^c \end{bmatrix} := \begin{bmatrix} V^a \bar{I}^a \\ V^b \bar{I}^b \\ V^c \bar{I}^c \end{bmatrix}$$

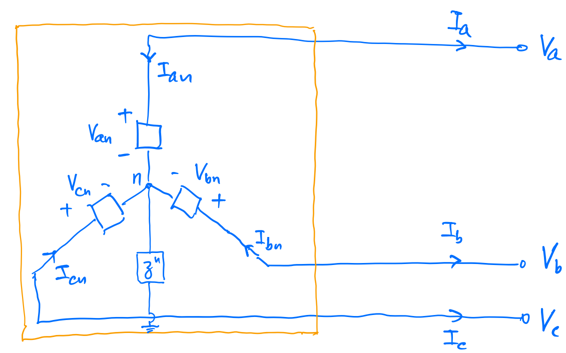


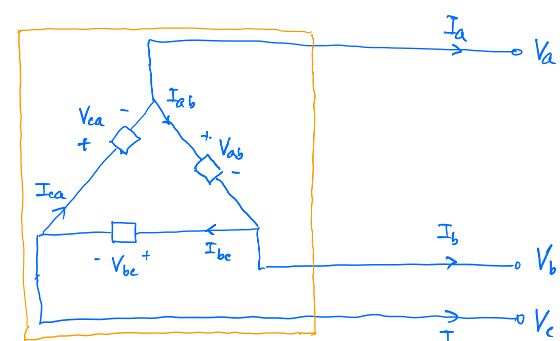
- V is with respect to an arbitrary common reference point, e.g. the ground
- I and s are in the direction out of the device



Internal vs external model

1. External model = Internal model + Conversion rule
    - External model: relation between (V, I, s)
    - Devices interact over network only through their terminal vars
2. Internal model : relation between $\left(V^{Y/\Delta}, I^{Y/\Delta}, s^{Y/\Delta}\right)$
    - Independent of Y or Δ configuration
    - Depends only on behavior of single-phase devices
    - Voltage/current/power source, impedance
3. Conversion rule : converts between internal and terminal vars
    - Depends only on Y or Δ configuration
    - Independent of type of single-phase devices



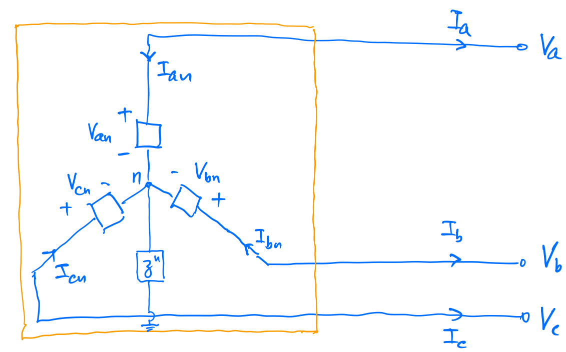


Conversion rule

Y configuration

Converts between internal and terminal variables

$$V = V^Y + V^n \mathbf{1}, \qquad I = -I^Y, \qquad s = -\left(s^Y + V^n \bar{I}^Y\right)$$



Device may or may not be grounded, and neutral impedance z^n may or may not be zero

Special case: if $V^n = 0$, then $V = V^Y$, $I = -I^Y$

Conversion rule

Δ configuration

Convert between internal vars and external vars

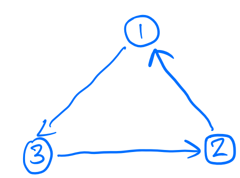
$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}}_{\Gamma} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}, \qquad \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = - \underbrace{\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}}_{\Gamma^{\mathsf{T}}} \begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \end{bmatrix}$$

In vector form

$$V^{\Delta} = \Gamma V, \qquad I = -\Gamma^{\mathsf{T}} I^{\Delta}$$

V^{Δ}: internal voltage; V: terminal voltage; I: terminal current; I^{Δ}: internal current

Γ is incidence matrix of:



Conversion matrices

Fortescue matrix F

Spectral decomposition:

$$\Gamma \;=\; F\Lambda\overline{F}, \qquad \Gamma^{\mathsf{T}} \;=\; \overline{F}\Lambda F$$

where

$$\Lambda := \begin{bmatrix} 0 & & \\ & 1-\alpha & \\ & & 1-\alpha^2 \end{bmatrix}, \qquad F := \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$$

eigenvectors of $\Gamma, \Gamma^{\mathsf{T}}$

positive-seq balanced vector α_+

negative-seq balanced vector α_-

and $\alpha := e^{-i2\pi/3}$

Pseudo-inverses: $\Gamma^{\dagger} \;=\; \frac{1}{3}\Gamma^{\mathsf{T}}, \qquad \Gamma^{\mathsf{T}\dagger} \;=\; \frac{1}{3}\Gamma$

Conversion rule

Δ configuration

1. Converts between internal and terminal voltages & currents

$$V^{\Delta} \;=\; \Gamma V, \qquad\qquad I \;=\; -\,\Gamma^{\mathsf{T}} I^{\Delta}$$

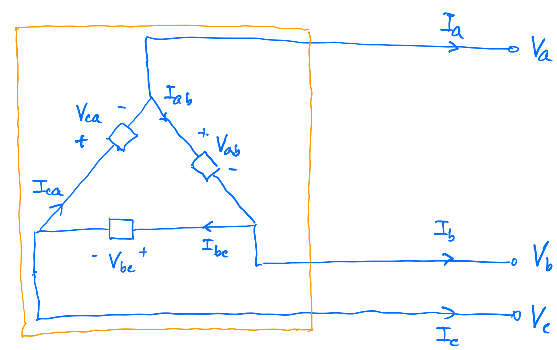
2. Given V^{Δ}: terminal voltage $V \;=\; \frac{1}{3}\Gamma^{\mathsf{T}} V^{\Delta} \;+\; \gamma\,1, \qquad \gamma \in \mathbb{C}$
   - $\gamma := \frac{1}{3} 1^T V$: zero-sequence terminal voltage (fixed by reference voltage)

3. Given I: internal current $I^{\Delta} \;=\; -\frac{1}{3}\Gamma I \;+\; \beta\,1, \qquad \beta \in \mathbb{C}$
   - $\beta := \frac{1}{3} 1^T I^{\Delta}$: zero-sequence internal current (does not affect terminal current)

4. Relation between s and s^{Δ} through $\left(V, I^{\Delta}\right)$:

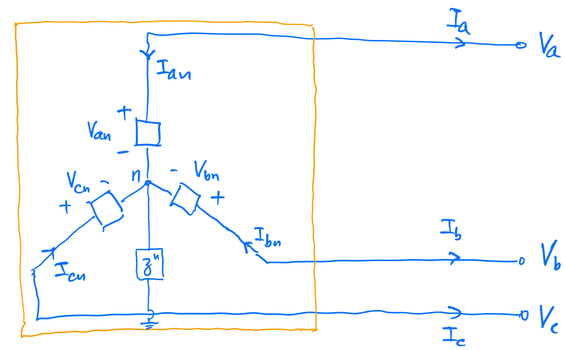
$$s \;=\; -\,\text{diag}\left(V I^{\Delta \mathsf{H}} \Gamma\right), \qquad s^{\Delta} \;=\; \text{diag}\left(\Gamma V I^{\Delta \mathsf{H}}\right)$$

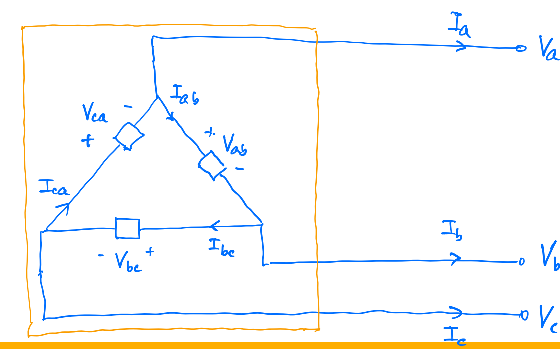
(no direct relation between s and s^{Δ})



3-phase device models

1. External model = Internal model + Conversion rule
    - External model: relation between (V, I, s)
    - Internal model: relation between $\left(V^{Y/\Delta}, I^{Y/\Delta}, s^{Y/\Delta}\right)$
2. Both internal and external models depend on device type
    - Voltage source
    - Current source
    - Power source
    - Impedance
3. … in Y and Δ configurations





Voltage source (E^Δ, z^Δ): Δ configuration

External model

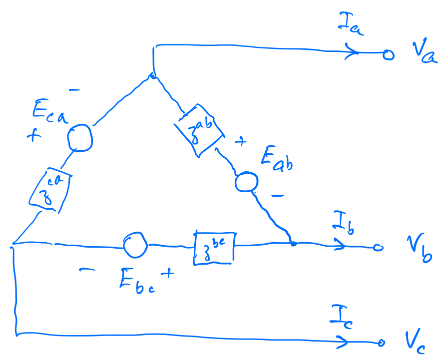
1. Internal model

$$V^\Delta \;=\; E^\Delta \;+\; z^\Delta I^\Delta$$

independent of Y/Δ config

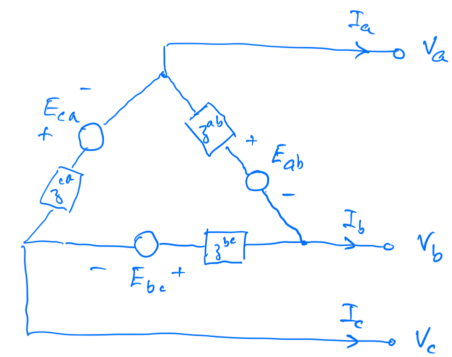
2. Conversion rule for Δ configuration

$$V^\Delta \;=\; \Gamma V, \qquad\qquad I \;=\; -\Gamma^\mathsf{T} I^\Delta$$



Voltage source $\left(E^{\Delta}, z^{\Delta}\right)$: Δ configuration

External model



1. Internal model

$$V^{\Delta} \;=\; E^{\Delta} + z^{\Delta} I^{\Delta}$$ independent of Y/Δ config

2. Conversion rule for Δ configuration

$$V^{\Delta} \;=\; \Gamma V, \qquad\qquad I \;=\; -\,\Gamma^{\mathsf{T}} I^{\Delta}$$

3. Two (asymmetric) relations between terminal vars (V, I)
   - Given V, 1st relation uniquely determines I (hence $\left(V^{\Delta}, I^{\Delta}\right)$ as well)
   - Given I, 2nd relation determines V up to zero-sequence voltage γ

Asymmetry is because V contains more info (γ) than I does (which contains no info about zero-sequence current $\beta := \frac{1}{3} 1^{\mathsf{T}} I^{\Delta}$)

Voltage source $\left(E^{\Delta}, z^{\Delta}\right)$: Δ configuration

External model

1. Given V,

$$I = \left(\Gamma^{\mathsf{T}} y^{\Delta}\right) E^{\Delta} - Y^{\Delta} V$$

$$Y^{\Delta} := \Gamma^{\mathsf{T}} y^{\Delta} \Gamma = \begin{bmatrix} y^{ab} + y^{ca} & -y^{ab} & -y^{ca} \\ -y^{ab} & y^{ab} + y^{bc} & -y^{bc} \\ -y^{ca} & -y^{bc} & y^{ca} + y^{bc} \end{bmatrix}, \quad y^{\Delta} := \left(z^{\Delta}\right)^{-1}$$

Voltage source $\left(E^\Delta, z^\Delta\right)$: Δ configuration
External model

1. Given V,

$$I \;=\; \left(\Gamma^\mathsf{T} y^\Delta\right) E^\Delta \;-\; Y^\Delta V$$

$$Y^\Delta \;:=\; \Gamma^\mathsf{T} y^\Delta \Gamma \;=\; \begin{bmatrix} y^{ab}+y^{ca} & -y^{ab} & -y^{ca} \\ -y^{ab} & y^{ab}+y^{bc} & -y^{bc} \\ -y^{ca} & -y^{bc} & y^{ca}+y^{bc} \end{bmatrix}, \quad y^\Delta := \left(z^\Delta\right)^{-1}$$

2. Given I with $1^\mathsf{T} I = 0$,

$$V \;=\; \hat{\Gamma} E^\Delta \;-\; Z^\Delta I \;+\; \gamma 1, \qquad 1^\mathsf{T} I \;=\; 0$$

$$\hat{\Gamma} \;:=\; \frac{1}{3}\Gamma^\mathsf{T}\left(\mathbb{I} - \frac{1}{\zeta}\,\tilde{z}^\Delta\, 1^\mathsf{T}\right), \qquad Z^\Delta \;:=\; \frac{1}{9}\Gamma^\mathsf{T} z^\Delta \left(\mathbb{I} - \frac{1}{\zeta}\, 1\, \tilde{z}^{\Delta\mathsf{T}}\right)\Gamma$$

Voltage source $\left(E^{\Delta}, z^{\Delta}\right)$: Δ configuration

External model

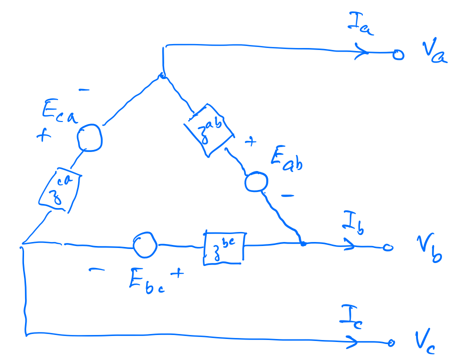
Comparison

Single-phase : $V = \ E - zI$

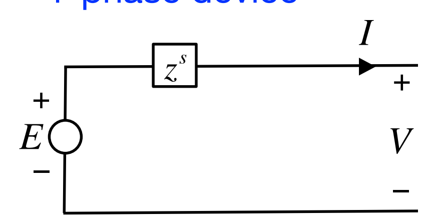
Three-phase : $V \ = \ \hat{\Gamma} E^{\Delta} \ - \ Z^{\Delta} I \ + \ \gamma 1, \qquad 1^{\mathsf{T}} I \ = \ 0$

rotated
internal voltage

voltage drop due to
equivalent impedance



1-phase device



Current source $\left(J^{\Delta}, y^{\Delta}\right)$: Δ configuration

External model

1. Internal model

$$I^{\Delta} \;=\; J^{\Delta} \;+\; y^{\Delta}\, V^{\Delta}$$

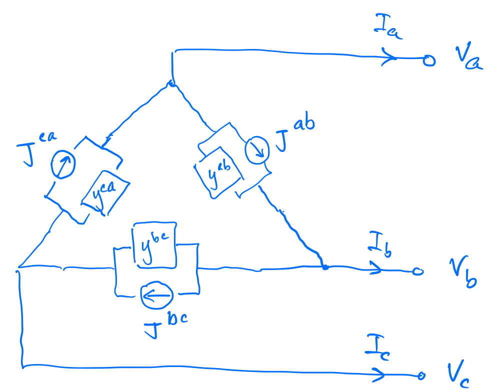
2. Conversion rule

$$V^{\Delta} \;=\; \Gamma V, \qquad\qquad I \;=\; -\,\Gamma^{\mathsf{T}} I^{\Delta}$$

3. $\Longrightarrow$ External model

$$I \;=\; -\left(\Gamma^{\mathsf{T}} J^{\Delta} \;+\; Y^{\Delta}\, V\right)$$

where (as before): $Y^{\Delta} \;:=\; \Gamma^{\mathsf{T}} y^{\Delta}\, \Gamma \;=\; \begin{bmatrix} y^{ab}+y^{ca} & -y^{ab} & -y^{ca} \\ -y^{ab} & y^{ab}+y^{bc} & -y^{bc} \\ -y^{ca} & -y^{bc} & y^{ca}+y^{bc} \end{bmatrix}$



Current source (J^Δ, y^Δ): Δ configuration

External model

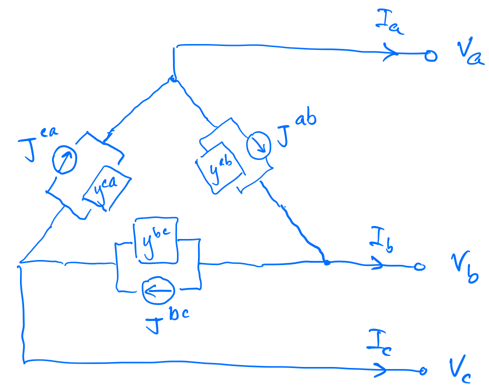
4. Comparison

Single-phase : $I \;=\; J - yV$

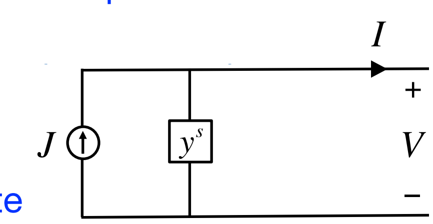
Three-phase :

$$I \;=\; -\Gamma^{\mathsf{T}} J^\Delta \;-\; Y^\Delta V$$

$$Y^\Delta \;:=\; \Gamma^{\mathsf{T}} y^\Delta \Gamma$$



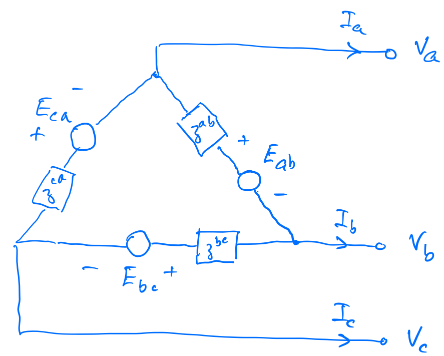
1-phase device



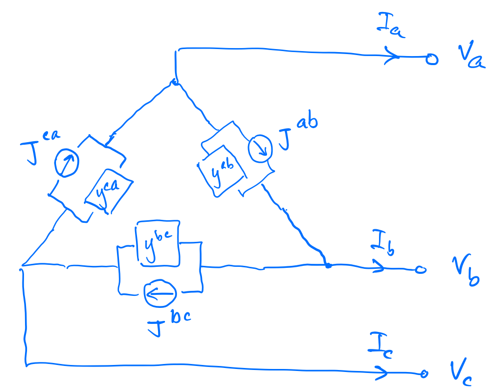
Note: directions of J are opposite

Voltage & current sources: comparison

1. Voltage source specifies E^{Δ} which does not uniquely determine terminal voltage V
   - $V \;=\; \hat{\Gamma} E^{\Delta} \;-\; Z^{\Delta} I \;+\; \gamma 1, \qquad 1^{\mathsf{T}} I \;=\; 0$
   - due to arbitrary zero-sequence voltage $\gamma := \frac{1}{3} 1^{\mathsf{T}} V$



2. Current source specifies J^{Δ} which uniquely determines terminal current I
   - $I \;=\; -\left(\Gamma^{\mathsf{T}} J^{\Delta} \;+\; Y^{\Delta} V\right)$
   - J^{Δ} contains its zero-sequence current $\beta := \frac{1}{3} 1^{\mathsf{T}} J^{\Delta}$



Impedance z^{Δ}: Δ configuration

External model

1. Internal model

$$V^{\Delta} \;=\; z^{\Delta} I^{\Delta}$$

2. Conversion rule

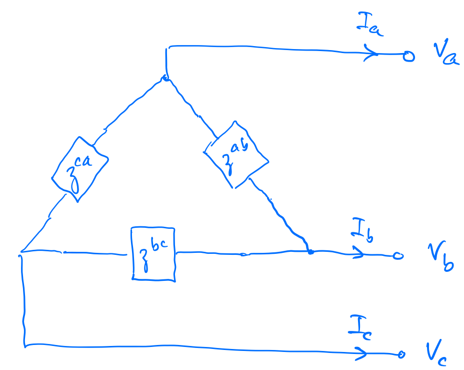
$$V^{\Delta} \;=\; \Gamma V, \qquad\qquad I \;=\; -\Gamma^{\mathsf{T}} I^{\Delta}$$

3. $\Longrightarrow$ External model

Given V, $I \;=\; -\,Y^{\Delta} V \;:=\; -\left(\Gamma^{\mathsf{T}} y^{\Delta} \Gamma\right) V$

Given I, $V \;=\; -\,Z^{\Delta} I \,+\, \gamma 1, \qquad 1^{\mathsf{T}} I \;=\; 0$

$$Z^{\Delta} \;:=\; \frac{1}{9}\Gamma^{\mathsf{T}} z^{\Delta}\left(\mathbb{I} - \frac{1}{\zeta} 1\, \tilde{z}^{\Delta\mathsf{T}}\right)\Gamma$$



As for voltage source, the asymmetry is because V contains more info (γ) than I does

Impedance z^{Δ}: Δ configuration

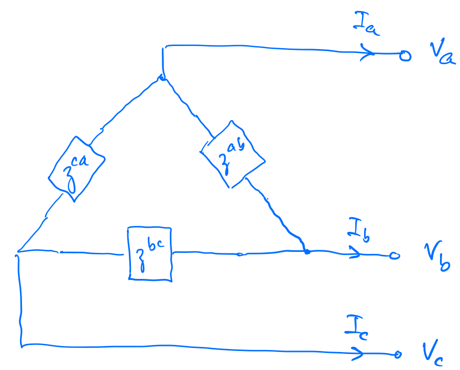
External model

4. Comparison

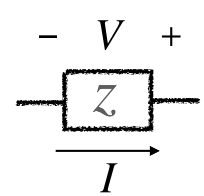
Single-phase : $V = -zI \quad \in \quad \mathbb{C}$

Three-phase : $V \;=\; -Z^{\Delta}I \;+\; \gamma 1, \qquad 1^{\mathsf{T}}I \;=\; 0$

voltage drop due to
equivalent impedance



1-phase device



Power source σ^Δ: Δ configuration

External model

1. Internal model

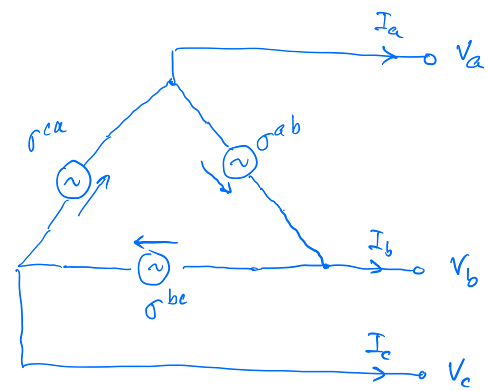
$$s^\Delta \;=\; \sigma^\Delta$$

2. Conversion rule

$$V^\Delta \;=\; \Gamma V, \qquad\qquad I \;=\; -\Gamma^{\mathsf{T}} I^\Delta$$

3. $\implies$ External model through $\left(V, I^\Delta\right)$

$$s \;=\; -\,\mathrm{diag}\left(V I^{\Delta \mathsf{H}} \Gamma\right), \quad \sigma^\Delta \;=\; \mathrm{diag}\left(\Gamma V I^{\Delta \mathsf{H}}\right)$$



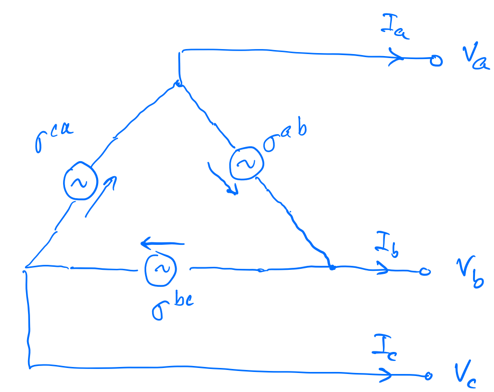
Power source σ^{Δ}: Δ configuration

External model

4. Comparison

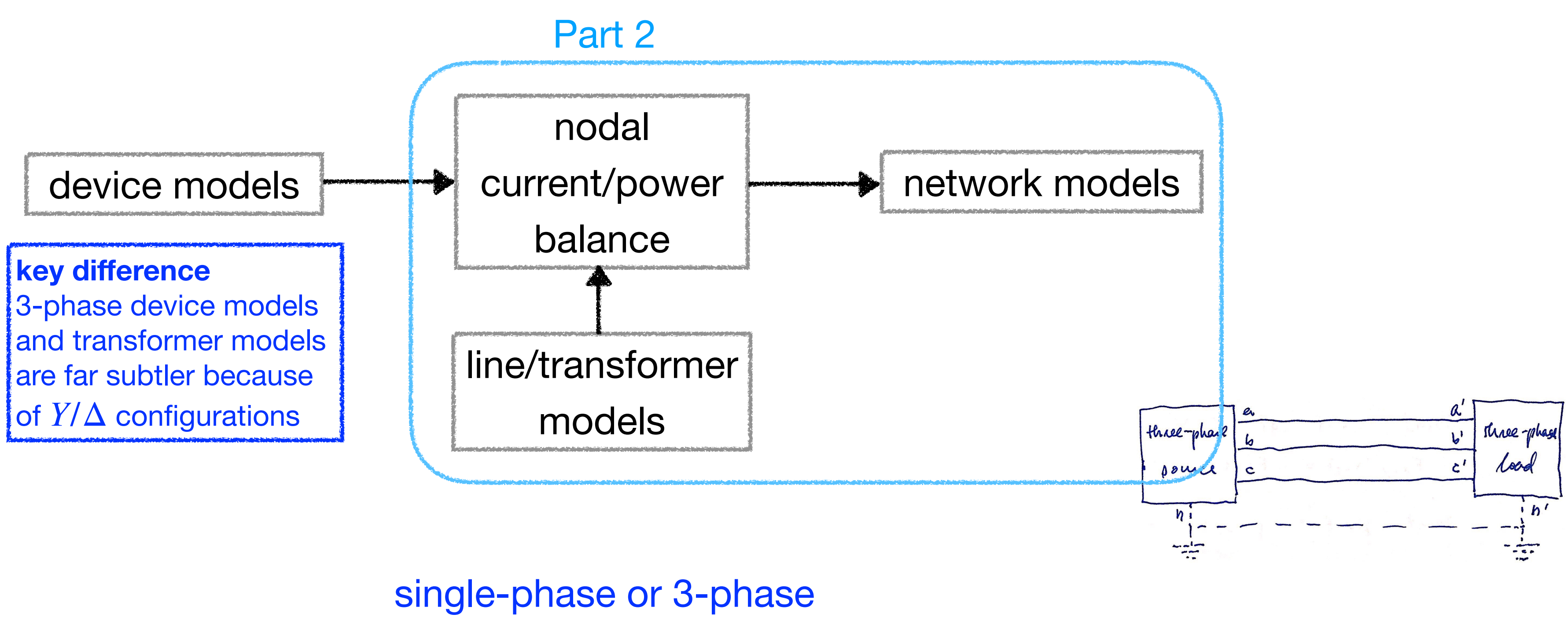
   Single-phase : $s = \sigma$

   Three-phase : $s = -\,\text{diag}\left(VI^{\Delta\mathsf{H}}\Gamma\right), \quad \sigma^{\Delta} = \text{diag}\left(\Gamma VI^{\Delta\mathsf{H}}\right)$



1-phase device

Overview

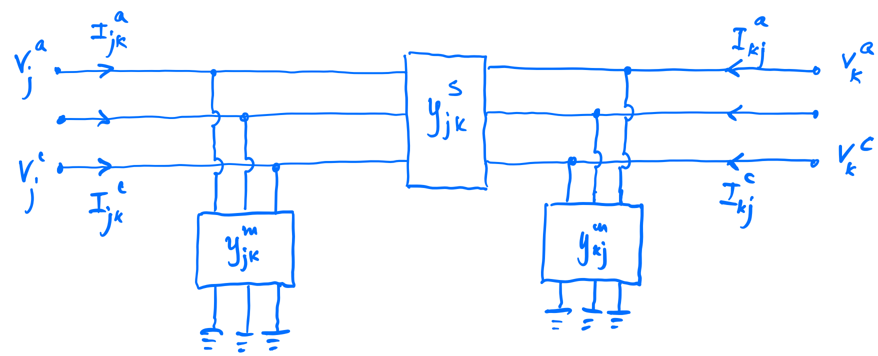

Part 2
device models
nodal current/power balance
network models
line/transformer models
key difference
3-phase device models and transformer models are far subtler because of Y/Δ configurations
three-phase source
three-phase load
a
b
c
a'
b'
c'
n
n'
single-phase or 3-phase

3-wire line model

With shunt admittances

Each line is characterized by

- Series admittance $y^s_{jk} := \left(z^s_{jk}\right)^{-1}$
- Shunt admittances $\left(y^m_{jk}, y^m_{kj}\right)$



Terminal voltages $\left(V_j, V_k\right)$ and terminal currents $\left(I_{jk}, I_{kj}\right)$ satisfy

$$I_{jk} = y^s_{jk}\left(V_j - V_k\right) + y^m_{jk}V_j$$

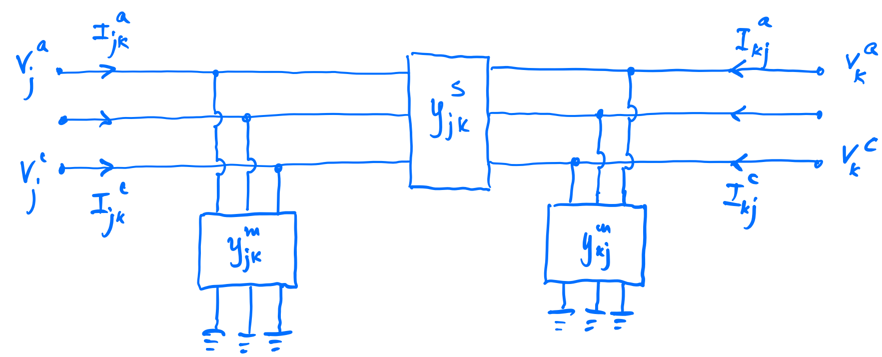
$$I_{kj} = y^s_{jk}\left(V_k - V_j\right) + y^m_{kj}V_k$$

3-wire line model

With shunt admittances

Each line is characterized by

- Series admittance $y^s_{jk} := \left(z^s_{jk}\right)^{-1}$
- Shunt admittances $\left(y^m_{jk}, y^m_{kj}\right)$



Terminal voltages $\left(V_j, V_k\right)$ and terminal power $\left(S_{jk}, S_{kj}\right)$ satisfy

$$S_{jk} := V_j\left(I_{jk}\right)^{\mathsf{H}} = V_j\left(V_j - V_k\right)^{\mathsf{H}}\left(y^s_{jk}\right)^{\mathsf{H}} + V_jV_j^{\mathsf{H}}\left(y^m_{jk}\right)^{\mathsf{H}}$$

$$S_{kj} := V_k\left(I_{kj}\right)^{\mathsf{H}} = V_k\left(V_k - V_j\right)^{\mathsf{H}}\left(y^s_{jk}\right)^{\mathsf{H}} + V_kV_k^{\mathsf{H}}\left(y^m_{kj}\right)^{\mathsf{H}}$$

Network equation

Nodal current balance

Series and shunt admittances

- 1-phase : scalars
- 3-phase : 3×3 (3-wire) or 4×4 (4-wire) matrices

3-phase sending-end currents:

$$I_{jk} = y_{jk}^s \left(V_j - V_k \right) + y_{jk}^m V_j, \qquad I_{kj} = y_{jk}^s \left(V_k - V_j \right) + y_{kj}^m V_k$$

Network equation

Nodal current balance

Series and shunt admittances

- 1-phase : scalars
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3-phase sending-end currents:

$$I_{jk} = y^s_{jk}\left(V_j - V_k\right) + y^m_{jk} V_j, \qquad I_{kj} = y^s_{jk}\left(V_k - V_j\right) + y^m_{kj} V_k$$

Nodal current balance:

$$I_j = \sum_{k:j\sim k} I_{jk} = \sum_{k:j\sim k} y^s_{jk}(V_j - V_k) + \left(\sum_{k:j\sim k} y^m_{jk}\right) V_j$$

$$= \left(\left(\sum_{k:j\sim k} y^s_{jk}\right) + y^m_{jj}\right) V_j - \sum_{k:j\sim k} y^s_{jk} V_k \qquad y^m_{jj} := \sum_{k:j\sim k} y^m_{jk}$$

Network equation

Nodal current balance

In terms of $3(N+1) \times 3(N+1)$ admittance matrix Y

$$I \;=\; Y\,V$$

$3(N+1)$ vector

where

$$Y_{jj} := \sum_{k:j\sim k} y^s_{jk} \;+\; y^m_{jj}$$

3×3 matrices

$$Y_{jk} := -\,y^s_{jk}$$

3×3 matrices

$$y^m_{jj} \;:=\; \sum_{k:j\sim k} y^m_{jk}$$

Y is complex (block-) symmetric [if network contains no 3-phase transformers in ΔY nor $Y\Delta$ confg]

It is admittance matrix of single-phase equivalent

Network equation

Nodal power balance

Nodal power balance

$$s_j \;=\; \sum_{k:j\sim k} \text{diag}\left(V_j(V_j - V_k)^H \left(y_{jk}^s\right)^H \;+\; V_j V_j^H \left(y_{jk}^m\right)^H \right)$$

$s_j = \text{diag}\left(V_j I_j^{\mathsf{H}}\right)$

generalizes single-phase:

$$s_j = \sum_{k:j\sim k} \left(|V_j|^2 - V_j V_k^H \right) \left(y_{jk}^s\right)^H \;+\; |V_j|^2 \left(y_{jj}^m\right)^H$$

Overall model

Device + network

1. Network model relates terminal vars (V, I, s)
   - Nodal current balance (linear): $I = YV$
   - Nodal power balance (nonlinear): $s_j = \sum_{k:j\sim k} \text{diag}\left(V_j(V_j - V_k)^{\mathsf{H}} y_{jk}^{s\mathsf{H}} \; + \; V_j V_j^{\mathsf{H}} y_{jk}^{m\mathsf{H}} \right)$
   - Either can be used
2. Device model for each 3-phase device
   - Internal model $\left(V_j^{Y/\Delta}, I_j^{Y/\Delta}, s_j^{Y/\Delta}, \gamma_j, \beta_j \right)$ + conversion rules
   - External model $\left(V_j, I_j, s_j, \gamma_j, \beta_j \right)$ with internal parameters
   - Either can be used
   - Power source models are nonlinear; other devices are linear

General 3-phase analysis

Buses j	Specification
N_v^Y	$V_j^Y := E_j^Y,\ \gamma_j$
N_v^Δ	$V_j^\Delta := E_j^\Delta,\ \gamma_j, \beta_j,$
N_c^Y	$I_j^Y := J_j^Y,\ \gamma_j$
N_c^Δ	$I_j^\Delta := J_j^\Delta$
N_i^Y	$z_j^Y,\ \gamma_j$
N_i^Δ	$z_j^\Delta,\ \beta_j$
N_p^Y	$\sigma_j^Y,\ \gamma_j$
N_p^Δ	$\sigma_j^\Delta,\ \gamma_j$

Variables at bus j:

- External vars : $(V_j, I_j, s_j), \gamma_j$
- Internal vars : $\left(V_j^{Y\Delta}, I_j^{Y/\Delta}, s_j^{Y/\Delta}\right), \beta_j$

Given: 3-phase devices & their specifications

- Voltage/current/power sources, impedances
- … in Y/Δ configuration

Calculate: remaining variables

Solution:

- Write down device+network model
- Solve numerically

General 3-phase optimization

Buses j	Specification
N_v^Y	$V_j^Y := E_j^Y,\ \gamma_j$
N_v^Δ	$V_j^\Delta := E_j^\Delta,\ \gamma_j, \beta_j,$
N_c^Y	$I_j^Y := J_j^Y,\ \gamma_j$
N_c^Δ	$I_j^\Delta := J_j^\Delta$
N_i^Y	$z_j^Y,\ \gamma_j$
N_i^Δ	$z_j^\Delta,\ \beta_j$
N_p^Y	$\sigma_j^Y,\ \gamma_j$
N_p^Δ	$\sigma_j^\Delta,\ \gamma_j$

Variables at bus j:

- External vars : $(V_j, I_j, s_j), \gamma_j$
- Internal vars : $\left(V_j^{Y\Delta}, I_j^{Y/\Delta}, s_j^{Y/\Delta}\right), \beta_j$

Given: 3-phase devices & uncontrollable quantities

- Voltage/current/power sources, impedances
- … in Y/Δ configuration

Min: cost (controllable variables & state)

Solution:

- Write down device+network model
- Write down additional constraints
- Solve numerically

Unbalance 3-phase modeling

Power System Analysis

A Mathematical Approach

Steven H. Low

DRAFT available at: http://netlab.caltech.edu/book/

Corrections, questions, comments appreciated!